NASA Contractor Report 1[illegible]

DEKFIS

~~DEFKI~~S USER'S GUIDE
DISCRETE EXTENDED KALMAN FILTER/SMOOTHER
PROGRAM FOR AIRCRAFT AND ROTORCRAFT DATA
CONSISTENCY

(NASA-CR-159081) DEKFIS USER'S GUIDE: DISCRETE EXTENDED KALMAN FILTER/SMOOTHER PROGRAM FOR AIRCRAFT AND ROTORCRAFT DATA CONSISTENCY (Systems Control, Inc., Palo Alto, Calif.) 69 p HC A04/MF A01 CSCL 09B G3/61

N81-22722

Unclas
42102

SYSTEMS CONTROL, INC. (Vt)
1801 Page Mill Road
Palo Alto, California 94304

NASA Contract NAS1-14549
November 1979

National Aeronautics and
Space Administration

Langley Research Center
Hampton Virginia 23665

TABLE OF CONTENTS

I. INTRODUCTION

System identification technology has been used successfully for many vehicles. Because of their large number of degrees of freedom and complex aerodynamic interactions, rotorcraft have always presented a special challenge to system identification methods. A completely integrated methodology has been developed under this NASA contract to solve this difficult problem. This methodology has also been translated into a user-oriented series of computer programs. This volume provides basic guidelines for efficient and effective use of one of these computer programs.

Figure 1.1 shows a schematic flowchart of the overall data processing technique for rotorcraft. The first step in this procedure is state estimation and instrument calibration. This is implemented by the computer program DEKFIS (Discrete Extended Kalman Filter and Smoother) which implements an extended Kalman filter/smoother using the Friedland-Duffy formulation. Instrument biases and scale factors are estimated at this stage, together with any state which is not measured directly. The second step involves estimation of the mathematical model of various forces, moments and interchanges. This is implemented in OSR (Optimal Subset Regression) computer program which uses a regression technique. Accurate estimates of parameters are obtained in the final step. One of two computer programs is used for this purpose. SCIDNT implements the maximum likelihood method for linear systems and NLSCIDNT extends the method to nonlinear rotorcraft models.

Accuracy of parameter estimates may be improved by using flight test inputs based on the input design program, INDES.

DEKFIS is a discrete extended Kalman filter/smoother program formulated for aircraft and helicopter state estimation and data consistency. DEKFIS is currently set up to pre-process raw test data by removing biases, correcting scale factor errors and

---

The contract research effort which led to the results in this report was financially supported by the Structures Laboratory, USARTL (AVRADCOM), NASA Langley Research Center and NASA Ames Research Center.

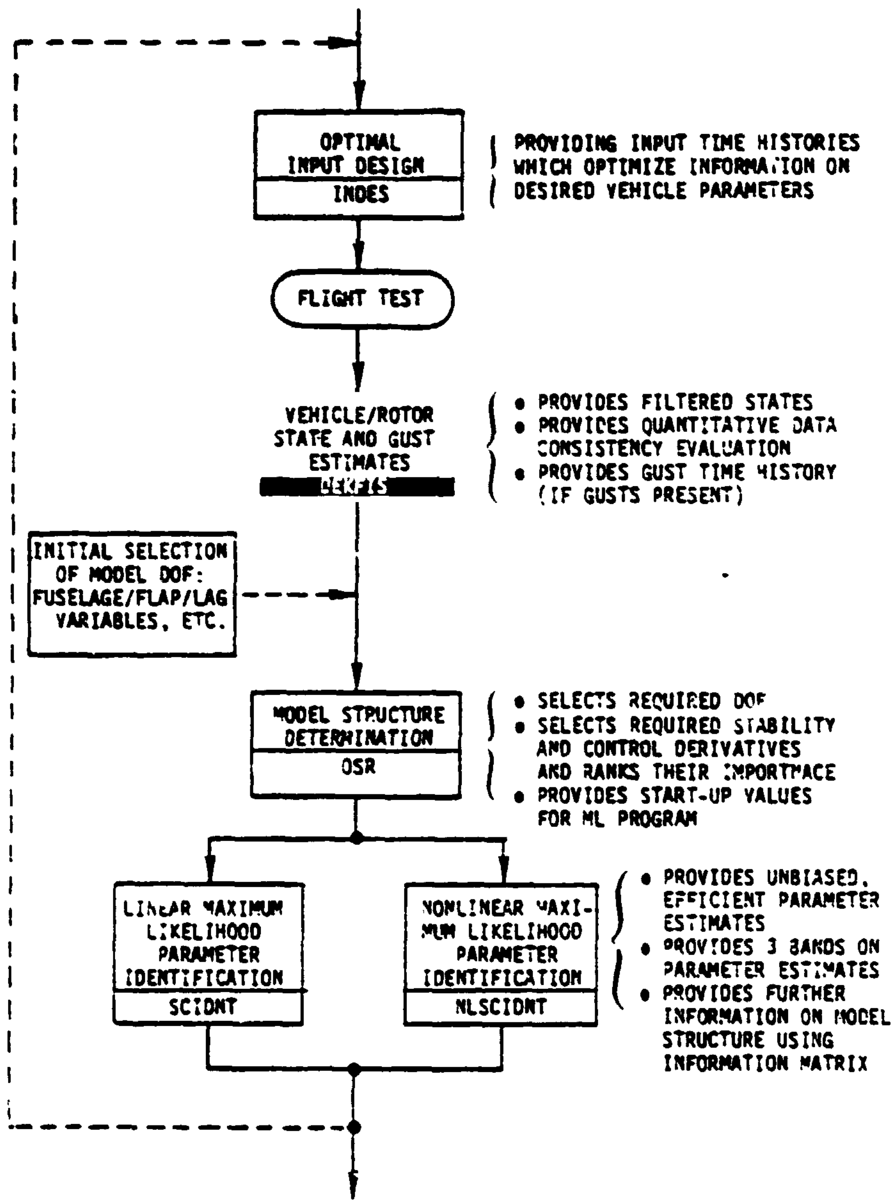


Figure 1.1 Integrated Rotorcraft System Identification Procedure

providing consistency with the aircraft inertial and kinematic equations. Hence, in its present formulation, DEKFIS will tend to suppress errors introduced by the measurement system and its dynamics, without affecting the overall system dynamics. Because of this, it is possible to separate the program into three separate estimators: the fuselage/gust estimator, the rotor state estimator and the RSRA estimator. This not only reduces the overall dimensionality and computational burden, but provides additional flexibility as shown in Table 1.1.

Table 1.1
Options

	FUSELAGE/GUST ESTIMATOR	ROTOR STATE ESTIMATOR	RSRA ESTIMATOR
FIXED WING AIRCRAFT	✓		
SINGLE ROTOR HELCIOPTER	✓	✓	
MULTIROTOR HELICOPTER	✓	Repeat for each rotor	
RSRA-FIXED WING CONFIG.	✓		✓
RSRA-COMPOUND CONFIG.	✓	✓	✓
WHIRL STAND DATA		✓	

The overall estimator has the same basic estimation algorithm for all the estimator blocks--only the subroutines containing the state and measurement equations and their partial derivatives change. The algorithm is a Friedland-Duffy extended Kalman filter with a locally iterated smoother for the filter pass, and a fixed interval smoother to provide gusts, process noise and initial conditions when desired.

II. PROGRAM DESCRIPTION

2.1 PROGRAM STRUCTURE

The overall logic of the DEKFIS program is presented in Figure 2.1. It is intended to give the user a general overview of the program. The subroutine calling structure is given in Figure 2.2. The user will find this figure useful in further understanding the program flow and in constructing an overlay structure should one be needed.

The functions of the more important subroutines are described below.

DEKFIS is the main routine. It performs no calculations, but calls four subroutines--INPUT, DIRCTR, FIS, and OUTPUT--which accomplish the computational and input/output tasks. DEKFIS sets up the maximum dimensions of the major arrays, and passes the appropriate STATE and MEAS subroutine addresses to DIRCTR and FIS, depending on whether the fuselage/gust estimator, the rotor state estimator, or the RSRA state estimator is being executed.

BLOCKD is a block data routine. Its major function is to initialize arrays which contain labels for the primary states, secondary states, measurements and noise sources for input/output purposes.

INPUT handles all the program's card and mass storage input, checks for errors in the input, initializes various flag arrays, and prints the title page and a summary of this information.

INREAD is called by input to read the time histories of the measurement and control variables.

PRTDAT prints out the measurement and control time histories.

SETDIM sets the dimensions and pointers for the component matrices of the D and R arrays. The dimensions are for the compressed matrices and vectors for the problem currently being executed.

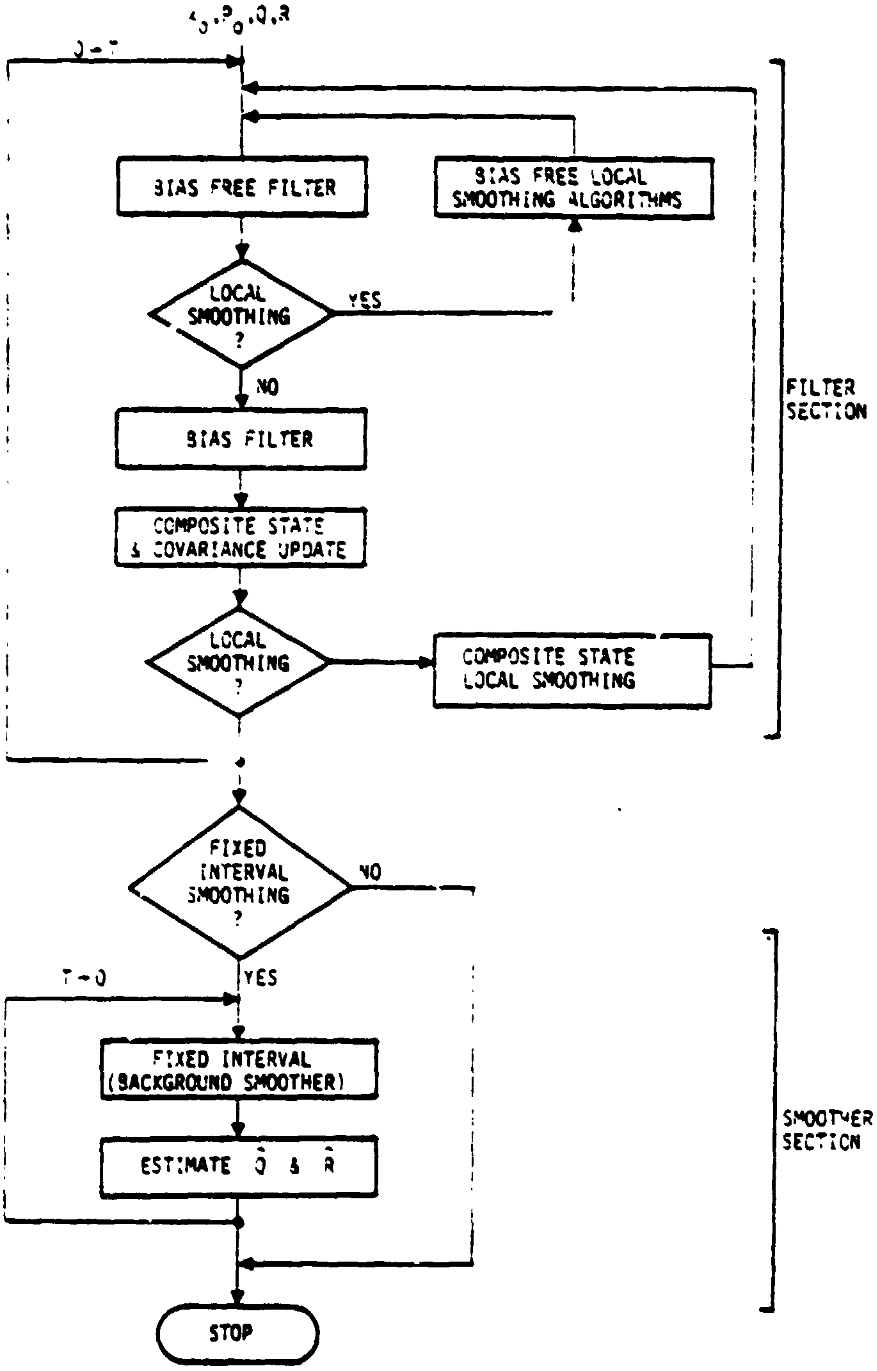
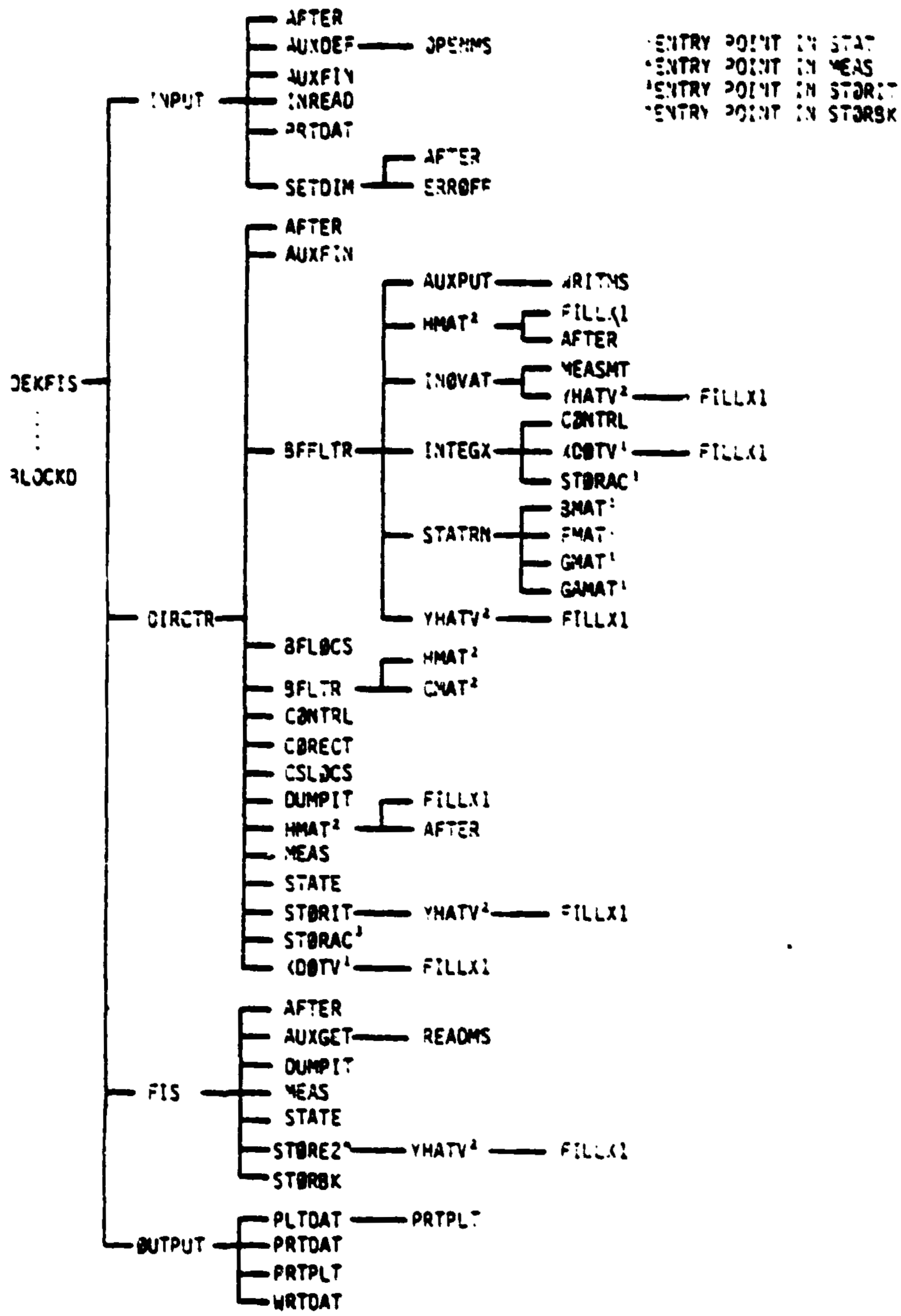


Figure 2.1 Filter/Smoother Algorithm



AUXILIARY ROUTINES:

ASPERR	CDIV1	EQUATE	MULTD2	ROTITL	UNITY
ABAT	CØMPRS	HQR2	MULTRI	REARNG	XAEB
ADD	DECMP2	INV	MULTT1	SCALE	ZERØ
AXEB	EIGENV	LNCNT	MULTT2	SØLVE2	
BALANC	ELMBAK	MULT	PRNT	SUBT	
BALBAK	ELMHES	MULTD1	RDMAT	TRANP	

Figure 2.2 Subroutine Calling Structure

DIRCTR performs the main computational tasks for the discrete extended Kalman filter and locally iterated smoother portion of the program. Its major features are a bias free filter, a bias free local smoother, a bias filter, a bias correction subroutine and a composite state local smoother. DIRCTR performs the initialization of the STATE, MEAS and other routines. It then sequences through all the data points yielding the filter estimate at each point.

BFFLTR performs the bias free filter calculations. It is called at each time point. The bias free filter is simply an extended Kalman filter for the primary states alone (i.e., the secondary or bias states are held constant). Hence, it performs prediction and update calculations for the primary states and the associated covariance.

INØVAT is called by BFFLTR to calculate the innovations for a given time point.

STATRN is called by BFFLTR to calculate the state transition matrix for a given time point.

INTEGX is called by BFFLTR to integrate the state equation to the next time point for the primary state prediction.

STATE contains the state equations for the appropriate estimator. It has entry points XDOTV, FMAT, GMAT, GAMAT and BMAT. XDOTV contains the nonlinear state equations of motion which are integrated to set the next time point. FMAT calculates the partial derivatives of the nonlinear state equations with respect to the primary states, GMAT calculates partial derivatives with respect to the control inputs, GAMAT calculates partial derivatives with respect to the process noise variables, and BMAT calculates partial derivatives with respect to the secondary states (or biases).

MEAS contains the measurement equations for the appropriate estimator. It has entry points YHATV, HMAT, DMAT and CMAT. YHATV contains the nonlinear measurement equations. HMAT calculates the partial derivatives of the nonlinear measurement equations with respect to the primary states, DMAT calculates the partial derivatives with respect to the control inputs, and CMAT calculates the partial derivatives with respect to the secondary state.

FILLXI is used following either a STATE or MEAS subroutine entry point call to create a reduced order (i.e.,

compressed) vector for use in the subsequent filter/ smoother calculations.

BFLOCS performs the bias-free local smoothing computations. Local smoothing iterations are used in conjunction with BFFLTR to improve the linearization of the primary states in the bias-free extended Kalman filter calculations.

BFLTR performs the bias filter computations, where the secondary (or bias) state estimates and covariances are calculated.

CONTRL finds the values of the control inputs at any specified time by linear interpolation.

CORECT performs the correction to the bias free state estimate and covariance to account for the secondary state estimates and covariances. This results in the composite state estimate and covariance.

CSLOCS performs the composite state local smoothing computations. Backward local smoothing iterations are used in conjunction with BFFLTR, BFLTR and CORECT to improve the composite state estimate.

DUMPIT is an output diagnostic routine that dumps the D matrix.

STORIT is the routine that stores the filter output for subsequent printing and plotting.

FIS performs the fixed interval smoothing computations. This backward smoothing is used on the primary states to provide a smoothed state estimate given the entire data record. An estimate of the process noise is also calculated.

OUTPUT is the subroutine that controls all program output — the printed time history tabulations, printer plots and writing data to a tape or permanent disc. Calls subroutines WRTDAT, PRTDAT and PLTDAT.

WRTDAT writes time history data to a permanent mass storage file.

PRTDAT prints time history tabulations.

PLTDAT generates printer plots of the time histories.

2.2 KALMAN FILTER EQUATIONS

The DEKFIS program utilizes a Friedland-Duffy extended Kalman filter with locally iterated smoothing to provide filter estimates and a fixed interval smoothing algorithm to provide gusts and smoothed estimates. The system equations can be written as:

$$\dot{x}_1(t) = f(x_1, x_2, u, t) + \Gamma(t)w(t) \quad , \quad x_1(0) = x_{1_o} \tag{1}$$

$$\dot{x}_2(t) = 0 \quad , \quad x_2(0) = b_o \tag{2}$$

$$y(t) = h(x_1, x_2, u, t) + v(t) \tag{3}$$

$$E\{w(t)\} = \overline{w}(t) \qquad E\{[w(t)-\overline{w}(t)][w(\tau)-\overline{w}(\tau)]^T\} = Q(t)\delta(t-\tau) \tag{4}$$

$$E\{v(t)\} = \overline{v}(t) \qquad E\{[v(t)-\overline{v}(t)][v(\tau)-\overline{v}(\tau)]^T\} = R(t)\delta(t-\tau) \tag{5}$$

where x_1 is the primary state and x_2 is the secondary state. The nonlinear f and h equations are linearized at each time point (as per the extended Kalman filter), defining the following matrixes.

$$F = \frac{\partial f}{\partial x_1} \quad (n_1 \times n_1) \qquad \Phi = \exp\{F\Delta t\} \qquad H = \frac{\partial h}{\partial x_1} \quad (m \times n_1)$$

$$B = \frac{\partial f}{\partial x_2} \quad (n_1 \times n_2) \qquad\qquad C = \frac{\partial h}{\partial x_2} \quad (m \times n_2)$$

$$\Gamma_D = F^{-1}(\Phi - I)\Gamma \qquad (n_1 \times P)$$

Using the Friedland-Duffy approach, the filter equations can be written as sets of bias-free and bias filter equations.

Bias-Free Filter Equations:

(1) $$\tilde{M}_i = (\Phi\tilde{P}\,\Phi^T + \Gamma_D Q\,\Gamma_D{}^T)_{i-1}$$

(2) $$\overline{x}_i = \int_{t_{i-1}}^{t_i} f(\hat{x}_{i-1}, b_{i-1}, u_{i-1}, t_{i-1})dt + \hat{x}_{i-1}$$

(3) $\tilde{W}_i = (H\tilde{M}H^T + R)_i$

(4) $K_{\bar{x}_i} = (\tilde{M}H^T\tilde{W}^{-1})_i$

(5) $\tilde{P}_i = [I - K_x H]_i \tilde{M}_i$

(6) $\nu_i = y_i - h(\bar{x}_i, 0,, u_i, t_i)$

(7) $\hat{x}_i = \bar{x}_i + K_{x_i}\nu_i$

where $x = x_1$ and $b = x_2$

Bias Filter Equations:

(1) $U_i = \Phi_{i-1} V_{i-1} + F^{-1}_{i-1}[\Phi_{i-1} - I] B_{i-1}$

(2) $S_i = H_i U_i + C_i$

(3) $V_i = U_i - K_{x_i} S_i$

(4) $\hat{W}_i = \tilde{W}_i + S_i P_{b_{i-1}} S_i^T$

(5) $K_{b_i} = P_{b_{i-1}} S_i^T \hat{W}_i^{-1}$

(6) $P_{b_i} = [I - K_{b_i} S_i] P_{b_{i-1}}$

(7) $\hat{b}_i = \hat{b}_{i-1} + K_{b_i} (\nu_i - S_i \hat{b}_{i-1})$

Note that the primary state estimate at the i^{th} time point is denoted by $\hat{x}_i$ and the secondary state (or bias) estimate is denoted by $\hat{b}_i$. The bias filter also utilizes K_{x_i}, $\hat{W}_i$ and ν_i which are computed in the bias-free filter. These two sets are fully coupled by the composite state update which corrects the bias-free state estimate for the effects of the bias.

Composite State and Covariance Update:

(1) $$\hat{x}_i^c = \begin{bmatrix} I & V_i \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{x}_i \\ \hat{b}_i \end{bmatrix}$$

(2) $$P_i^c = \begin{bmatrix} P_x & P_{xb} \\ P_{xb}^T & P_b \end{bmatrix}_i$$

(3) $$P_{x_i} = \tilde{P}_i + V_i \, P_{b_i} \, V_i^T$$

(4) $$P_{xb_i} = V_i \, P_{b_i}$$

For the local smoothing options, it is possible to iterate on either the bias-free state estimates and/or the composite state filter estimates.

Bias-Free Local Smoothing:

(1) $$\hat{x}_{i-1|i} = \hat{x}_{i-1} + \tilde{P}_{i-1} \, \Phi_{i-1}^T \, [I - K H^T]_i^T \, H_i^T \, R^{-1} \nu_i$$

Composite State Local Smoothing:

(2) $$\hat{x}^c_{i-1|i} = \hat{x}^c_{i-1} + P^c_{i-1} \, \Phi^{c^T}_{i-1} \left\{ I - \begin{bmatrix} K_x \\ K_b \end{bmatrix} [H \,\vdots\, C] \right\}_i^T \begin{bmatrix} H^T \\ C^T \end{bmatrix}_i R^{-1} \nu$$

where

(3) $$\Phi^c = \begin{bmatrix} \Phi & F^{-1}\{\Phi - I\}B \\ 0 & I \end{bmatrix}$$

The fixed interval smoothing equations are used when the filter estimates are not adequate. These equations are necessary when some states are driven by process noise (i.e., gust states) and it is necessary to estimate this process noise. A further benefit of fixed interval smoothing is that smoothing estimates are computed for all the primary states. These equations are shown on the following page.

Fixed Interval Smoothing:

(1) $\hat{x}_{i\,N} = \hat{x}_i - P_i \phi_i^T \lambda_i$

(2) $\hat{w}_{i\,N} = \bar{w}_i - Q_i \Gamma_i^T \lambda_i$

(3) $\lambda_{i-1} = (I - P_i H_i^T R_i^{-1} H_i)^T [\phi_i^T \lambda_i - H_i R_i^{-1} (z_i - H_i \bar{x}_i)], \ \lambda_N = 0$

2.3 STATE AND MEASUREMENT EQUATIONS

The DEKFIS program has the capability of handling any non-linear state and measurement equations that are programmed in the STATE and MEAS subroutines. The rotorcraft version of DEKFIS has the equations for three independent estimators: the fuselage/gust estimator, the rotor state estimator and the RSRA estimator. These equations are discussed below.

2.3.1 Fuselage/Gust Estimator Equations

The uncompressed state and measurement vectors for the fuselage/gust estimator are shown in Table 2.1. The user has the option of choosing which states (and hence which equations are to be integrated), which measurements and which process noise sources are to be used for a particular problem.

Table 2.1

Fuselage/Gust Estimator-Vector Definitions (Uncompressed)

$$\underset{25\times 1}{\underline{x}_1} = \begin{bmatrix} \phi \\ \theta \\ \psi \\ v_x \\ v_y \\ v_z \\ p \\ q \\ r \\ a_{xI} \\ a_{yI} \\ a_{zI} \\ \dot{p}_I \\ \dot{q}_I \\ \dot{r}_I \\ V_{gN} \\ V_{gE} \\ V_{gV} \\ X_N \\ Y_E \\ Z_V \\ \text{dummy} \\ \text{dummy} \\ \text{dummy} \\ \text{dummy} \end{bmatrix}
\qquad
\underset{50\times 1}{\underline{x}_2} = \begin{bmatrix} b_\phi \\ b_\theta \\ b_\psi \\ b_{v_x} \\ b_{v_y} \\ b_{v_z} \\ b_p \\ b_q \\ b_r \\ b_{a_{xI}} \\ b_{a_{yI}} \\ b_{a_{zI}} \\ b_{\dot{p}_I} \\ b_{\dot{q}_I} \\ b_{\dot{r}_I} \\ b_{X_N} \\ b_{Y_E} \\ b_{Z_V} \\ b_V \\ b_\beta \\ b_\alpha \\ b_R \\ b_{\beta_R} \\ b_{\alpha_R} \\ k_\phi \end{bmatrix}
\overset{\text{cont'd}}{\begin{bmatrix} k_\theta \\ k_\psi \\ k_{v_x} \\ k_{v_y} \\ k_{v_z} \\ k_p \\ k_q \\ k_r \\ k_{a_{xI}} \\ k_{a_{yI}} \\ k_{a_{zI}} \\ k_{\dot{p}_I} \\ k_{\dot{q}_I} \\ k_{\dot{r}_I} \\ k_{X_N} \\ k_{Y_E} \\ k_{Z_V} \\ k_V \\ k_\beta \\ k_\alpha \\ k_R \\ k_{\beta_R} \\ k_{\alpha_R} \\ \text{dummy} \\ \text{dummy} \end{bmatrix}}$$

$$\underset{24\times 1}{\underline{y}} = \begin{bmatrix} \phi_m \\ \theta_m \\ \psi_m \\ v_{x_m} \\ v_{y_m} \\ v_{z_m} \\ p_m \\ q_m \\ r_m \\ a_{xI_m} \\ a_{yI_m} \\ a_{zI_m} \\ \dot{p}_{I_m} \\ \dot{q}_{I_m} \\ \dot{r}_{I_m} \\ X_{N_m} \\ Y_{E_m} \\ Z_{V_m} \\ V_m \\ \beta_m \\ \alpha_m \\ R_m \\ \beta_{R_m} \\ \alpha_{R_m} \end{bmatrix}
\begin{array}{l} \text{rad} \\ \text{rad} \\ \text{rad} \\ \text{ft/sec} \\ \text{ft/sec} \\ \text{ft/sec} \\ \text{rad/sec} \\ \text{rad/sec} \\ \text{rad/sec} \\ \text{ft/sec}^2 \\ \text{ft/sec}^2 \\ \text{ft/sec}^2 \\ \text{rad/sec}^2 \\ \text{rad/sec}^2 \\ \text{rad/sec}^2 \\ \text{ft} \\ \text{ft} \\ \text{ft} \\ \text{ft/sec} \\ \text{rad} \\ \text{rad} \\ \text{ft} \\ \text{rad} \\ \text{rad} \end{array}$$

$$\underset{9\times 1}{\underline{w}} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \\ w_9 \end{bmatrix}$$

The $\underline{x}_1$ vector is of size 25x1 and contains the primary states for the fuselage/gust estimator. These are: the aircraft attitudes - Euler ϕ, θ, ψ; the aircraft inertial velocities in the aircraft body axis - v_x, v_y, v_z; the fuselage angular rates—p, q, r; the linear accelerometer indicated acceleration — a_{x_I}, a_{y_I} a_{z_I} (this differs from the actual linear acceleration by a first order lag); the angular accelerometer indicated acceleration — $\dot{p}_I, \dot{q}_I, \dot{r}_I$ (likewise lagged); gust velocities in an inertial north, east, vertical frame — v_{g_N}, v_{g_E}, v_{g_V}; and the aircraft position in an inertial north, east, vertical frame — X_N, Y_E, Z_V.

The $\underline{y}$ vector is of size 24x1 and contains the measurements for the fuselage/gust estimator. These are: the aircraft attitudes — ϕ_m, θ_m, ψ_m; the aircraft airspeed in the aircraft body axis — $v_{x_m}, v_{y_m}, v_{z_m}$ (or either alternately or redundantly as V_m, β_m, α_m measurements); the aircraft angular rates —p_m, q_m, r_m; the linear accelerometer measurements — $a_{x_{I_m}}$, $a_{y_{I_m}}, a_{z_{I_m}}$; the angular accelerometer measurements — $\dot{p}_{I_m}$, $\dot{q}_{I_m}$, $\dot{r}_{I_m}$; and radar position measurements in an inertial north, east, vertical frame — X_{N_m}, Y_{E_m}, Z_{V_m} (or alternately R_m, β_{R_m}, α_{R_m}, the radar range, elevation and azimuth coordinates.). The associated units are to the right of this vector. (The actual measurement data input to the program should have these units.)

The $\underline{x}_2$ vector is of size 50x1 and contains the biases (ie b's) and scale factors (ie k's) for each of the measurements.

The $\underline{w}$ vector is a 9x1 process noise vector. This process noise drives the linear accelerations — w_1, w_2, w_3; the angular accelerations — w_4, w_5, w_6; and the gusts —w_7, w_8, w_9.

The $\underline{u}$ vector is a 4x1 control vector that is dummied up since there are no control inputs used in the fuselage/gust state equations.

The nonlinear fuselage/gust equations are presented below:

Primary State Equations

$$\dot{\phi} = p + (r \cos \phi + q \sin \phi) \tan \theta$$

$$\dot{\theta} = q \cos \phi - r \sin \phi$$

$$\dot{\psi} = (r \cos \phi + q \sin \phi)/\cos \theta$$

$$\dot{v}_x = v_y\, r - v_z\, q - g \sin \theta + g \sin \theta(o) + w_1 - w_5 \ell_{z1} + w_6 \ell_{y1}$$
$$+ (q^2 + r^2) \ell_{x1} - qp\ell_{y1} - rp\ell_{z1}$$

$$\dot{v}_y = v_z p - v_x + g \sin \phi \cos \theta - g \sin \phi(o) \cos\theta(o) + w_2 - w_6 \ell_{x2}$$
$$+ w_4 \ell_{z_2} - p\, q\, \ell_{x2} + (p^2 + r^2)\, \ell_{y2} - rq\ell_{z2}$$

$$\dot{v}_z = v_x\, q - v_y p + g \cos \phi \cos\theta - g \cos \phi(o) \cos\theta(o) + w_3 - w_4 \ell_{y3}$$
$$+ w_5 \ell_{x3} - pr\, \ell_{x3} - q\, r\, \ell_{y3} + (p^2 + q^2) \ell_{z3}$$

$$\dot{p} = w_4$$

$$\dot{q} = w_5$$

$$\dot{r} = w_6$$

$$\dot{a}_{xI} = \alpha_1 a_{xI} - \alpha_1 w_1$$

$$\dot{a}_{yI} = \alpha_2 a_{yI} - \alpha_2 w_2$$

$$\dot{a}_{zI} = \alpha_3 a_{zI} - \alpha_3 w_3$$

$$(\dot{p}_I)^{\cdot} = \alpha_4 \dot{p}_I - \alpha_4 w_4$$

$$(\dot{q}_I)^{\cdot} = \alpha_5 \dot{q}_I - \alpha_5 w_5$$

$$(\dot{r}_I)^{\cdot} = \alpha_6 \dot{r}_I - \alpha_6 w_6$$

$$\dot{V}_{g_N} = \alpha_7 V_{g_N} - \alpha_7 w_7$$

$$\dot{V}_{gE} = \alpha_8 V_{gE} - \alpha_8 w_8$$

$$\dot{V}g_v = \alpha_9 Vg_v - \alpha_9 w$$

$$\dot{X}_n = v_x \cos\theta \cos\psi + v_y \sin\theta \sin\phi \cos\psi - v_y \cos\theta \sin\psi + v_z \sin\theta \cos\psi \cos\theta + v_z \sin\phi \sin\psi$$

$$\dot{Y}_e = v_x \cos\theta \sin\psi + v_y \sin\theta \sin\phi \sin\psi + v_y \cos\phi \cos\psi + v_z \sin\theta \cos\phi \sin\psi - v_z \sin\phi \cos\psi$$

$$\dot{Z}_v = - v_x \sin\theta + v_y \cos\theta \sin\phi + v_z \cos\theta \cos\phi$$

Secondary State Equations

$$\dot{\underline{x}}_2 = \underline{0}$$

Measurement Equations

$$\phi_m = k_\phi \cdot \phi + b_\phi + n_1$$

$$\theta_m = k_\theta \cdot \theta + b_\theta + n_2$$

$$\psi_m = k_\psi \cdot \psi + b_\psi + n_3$$

$$v_{x_m} = k_{v_x} (v_x + v_{x_g}) + b_{v_x} + n_4$$

$$v_{y_m} = k_{v_y} (v_y + v_{y_g}) + b_{v_y} + n_5$$

$$v_{z_m} = k_{v_z} (v_z + v_{z_g}) + b_{v_z} + n_6$$

$$p_m = k_p \cdot p + b_p + n_7$$

$$q_m = k_q \cdot q + b_q + n_8$$

$$r_m = k_r \cdot r + b_r + n_9$$

$$a_{x_{I_m}} = k_{a_{x_I}} (a_{x_I} + g \sin \theta (0)) + b_{a_{x_I}} + n_{10}$$

$$a_{y_{I_m}} = K_{a_{y_I}} (a_{y_I} - g \sin \phi (0) \cos \theta(0)) + b_{a_{y_I}} + n_{11}$$

$$a_{z_{I_m}} = k_{a_{z_I}} (a_{z_I} - g \cos \phi(0) \cos\theta(0)) + b_{a_{z_I}} + n_{12}$$

$$\dot{p}_{I_m} = k_{\dot{p}_I} \cdot \dot{p}_I + b_{\dot{p}_I} + n_{13}$$

$$\dot{q}_{I_m} = k_{\dot{p}_I} \cdot \dot{q}_I + b_{\dot{q}_I} + n_{14}$$

$$\dot{r}_{I_m} = k_{\dot{r}_I} \cdot \dot{r}_I + b_{\dot{r}_I} + n_{15}$$

$$X_{N_m} = k_{x_n} \cdot XN + b_{x_N} + n_{16}$$

$$Y_{E_m} = k_{y_E} \cdot Y_E + b_{y_E} + n_{17}$$

$$z_{v_m} = k_{z_v} \cdot z_v + b_{z_v} + n_{18}$$

$$V_m = k_v \cdot ((v_x + v_{x_g} + q \ell_{z_4} - r \ell_{y_4})^2 + (v_y + v_{y_g} + \ell_{x_4} - p \ell_{z_4})^2$$

$$+ (v_z + v_{z_g} + p \ell_{y_4} - q \ell_{x_4})^2)^{1/2} + b_v + n_{19}$$

$$\beta_m = k_\beta \cdot \tan^{-1}((v_y + v_{y_g} + r \ell_{x_5} - p \ell_{z_5}) / (v_x + v_{x_g} + q \ell_{z_5} - r \ell_{y_5}))$$

$$+ b_\beta + n_{20}$$

$$\alpha_m = k_\alpha \cdot \tan^{-1}((v_z + v_{z_g} + p\ell_{y_6} - q\ell_{x_6})/(v_x + v_{x_g} + q\ell_{z_6} - r\ell_{y_6}))$$
$$+ b_\alpha + n_{21}$$

$$R_m = k_R \, (X_N^2 + Y_E^2 + Z_V^2)^{1/2} + b_R + n_{22}$$

$$\beta_{R_m} = k_{\beta_R} \cdot \tan^{-1}(-Z_V/(X_N^2 + Y_E^2)^{1/2}) + b_{\beta_R} + n_{23}$$

$$\alpha_{R_m} = k_{\alpha_R} \cdot \tan^{-1}(-Y_E/X_N) + b_{\alpha_R} + n_{24}$$

where,

$$V_{x_g} = (V_{g_N} + V_{w_N})\cos\theta\cos\psi + (V_{g_E} + V_{w_E})\cos\theta\sin\psi - (V_{g_V} + V_{w_V})\sin\theta$$

$$V_{y_g} = (V_{g_N} + V_{w_N})\sin\theta\sin\phi\cos\psi - (V_{g_N} + V_{w_N})\cos\theta\sin\psi$$
$$+ (V_{g_E} + V_{w_E})\sin\theta\sin\phi\sin\psi + (V_{g_E} + V_{w_E})\cos\phi\cos\psi$$
$$+ (V_{g_V} + V_{w_V})\cos\theta\sin\phi$$

$$V_{z_g} = (V_{g_N} + V_{w_N})\sin\theta\cos\phi\cos\psi + (V_{g_N} + V_{w_N})\sin\phi\sin\psi$$
$$+ (V_{g_E} + V_{w_E})\sin\theta\cos\phi\sin\psi - (V_{g_E} + V_{w_E})\sin\phi\cos\psi$$
$$+ (V_{g_V} + V_{w_V})\cos\theta\cos\phi$$

The parameters used in these equations are defined in the fuselage/gust estimator parameter list shown in Table 2.2. The terms n_i, $i = 1,24$ represent random measurement noise. All locations are defined in the aircraft body axis system -- positive forward, right, down from the aircraft center of gravity.

Table 2.2

Fuselage/Gust Estimator-Parameter List

PARAMETER INDEX	PARAMETER DESCRIPTION	
1-25	$\underline{x}_1$ ic's (use uncompressed vector)	
26-75	$\underline{x}_2$ ic's (use uncompressed vector)	
76	ℓ_{x1}	a_x accelerometer location, ft.(relative to c.g.)
77	ℓ_{y1}	
78	ℓ_{z1}	
79	ℓ_{x2}	a_y accelerometer location, ft
80	ℓ_{y2}	
81	ℓ_{z2}	
82	ℓ_{x3}	a_z accelerometer location, ft
83	ℓ_{y3}	
84	ℓ_{z3}	
85	ℓ_{x4}	V probe location, ft
86	ℓ_{y4}	
87	ℓ_{z4}	
88	ℓ_{x5}	β vane location, ft
89	ℓ_{y5}	
90	ℓ_{z5}	
91	ℓ_{x6}	α vane location, ft
92	ℓ_{y6}	
93	ℓ_{z6}	
94	α_1	accelerometer time constants
95	α_2	
96	α_3	
97	α_4	
98	α_5	
99	α_6	
100	α_7	gust power spectral time constants
101	α_8	
102	α_9	
103	V_{W_N}	steady gust components (wind)
104	V_{W_E}	
105	V_{W_V}	

2.3.2 Rotor State Estimator Equations

The uncompressed state and measurement vectors for the rotor state estimator are shown in Table 2.3. As in the previous estimator, the user has the option to flag which states, measurements and process noise sources he wants to use.

The $\underline{x}_1$ vector is a 14x1 vector of primary states used in the rotor state estimator. These are: rotor coning angle $-\beta o$; rotor longitudinal flapping $-\beta 1c$; rotor lateral flapping $-\beta 1s$; simular fixed system coordinates for the rotor lagging motion - ζo, $\zeta 1c$, $\zeta 1s$; rotor azimuth angle - ψ_R; and the time derivatives of each of these - $\dot{\beta} o$, $\dot{\beta} 1c$, $\dot{\beta} 1s$, $\dot{\zeta} o$, $\dot{\zeta} 1c$, $\dot{\zeta} 1s$, $\dot{\psi}_R$.

The $\underline{y}$ vector is a 13x1 vector of measurements and associated units for the rotor state estimator. These are: the individual blade flapping angles β_{im}, i = 1, 6; the individual blade lagging angles ζ_{im}, i = 1, 6; and the cosine of rotor azimuth $-\cos \psi_{R_m}$.

The $\underline{x}_2$ vector is a 26x1 vector of biases and scale factors for each of the aforementioned measurements.

The $\underline{w}$ vector is a 7x1 vector of process noise sources each driving one of the seven rotor degrees of freedom modelled.

The state and measurement equations for the rotor state estimator are presented below:

Primary State Equations

$$\dot{\underline{x}}_1 = \underline{F}\, \underline{x}_1 + \underline{\Gamma}\, \underline{w}$$

$$\underline{F} = \left[\begin{array}{cc} \underline{0} & \begin{matrix} 1.0 & & & & & & \\ & 1.0 & & & & & \\ & & 1.0 & & & & \\ & & & 1.0 & & & \\ & & & & 1.0 & & \\ & & & & & 1.0 & \\ & & & & & & 1.0 \end{matrix} \\ \hdashline \underline{0} & \underline{0} \end{array}\right] \qquad \underline{\Gamma} = \left[\begin{array}{c} \underline{0} \\ \hdashline \begin{matrix} 1.0 & & & & & & \\ & 1.0 & & & & & \\ & & 1.0 & & & & \\ & & & 1.0 & & & \\ & & & & 1.0 & & \\ & & & & & 1.0 & \\ & & & & & & 1.0 \end{matrix} \end{array}\right]$$

Table 2.3
Rotor State Estimator-Vector Definition (Uncompressed)

$$
\underset{14\times1}{\underline{x}_1} = \begin{bmatrix} \beta_0 \\ \beta_{1c} \\ \beta_{1s} \\ \zeta_0 \\ \zeta_{1c} \\ \zeta_{1s} \\ \psi_R \\ \dot{\beta}_0 \\ \dot{\beta}_{1c} \\ \dot{\beta}_{1s} \\ \dot{\zeta}_0 \\ \dot{\zeta}_{1c} \\ \dot{\zeta}_{1s} \\ \dot{\psi}_R \end{bmatrix}
\qquad
\underset{26\times1}{\underline{x}_2} = \begin{bmatrix} b\beta_1 \\ b\beta_2 \\ b\beta_3 \\ b\beta_4 \\ b\beta_5 \\ b\beta_6 \\ b\zeta_1 \\ b\zeta_2 \\ b\zeta_3 \\ b\zeta_4 \\ b\zeta_5 \\ b\zeta_6 \\ b_{\cos\psi_R} \\ k\beta_1 \\ k\beta_2 \\ k\beta_3 \\ k\beta_4 \\ k\beta_5 \\ k\beta_6 \\ k\zeta_1 \\ k\zeta_2 \\ k\zeta_3 \\ k\zeta_4 \\ k\zeta_5 \\ k\zeta_6 \\ k_{\cos\psi_R} \end{bmatrix}
\qquad
\underset{13\times1}{\underline{y}} = \begin{bmatrix} \beta_{1_m} \\ \beta_{2_m} \\ \beta_{3_m} \\ \beta_{4_m} \\ \beta_{5_m} \\ \beta_{6_m} \\ \zeta_{1_m} \\ \zeta_{2_m} \\ \zeta_{3_m} \\ \zeta_{4_m} \\ \zeta_{5_m} \\ \zeta_{6_m} \\ \cos\psi_{R_m} \end{bmatrix}
\begin{matrix} \text{rad} \\ \text{rad} \\ \text{rad} \\ \text{rad} \\ \text{rad} \\ \text{rad} \\ \text{rad} \\ \text{rad} \\ \text{rad} \\ \text{rad} \\ \text{rad} \\ \text{rad} \\ \text{N.D.} \end{matrix}
$$

$$
\underset{7\times1}{\underline{w}} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \end{bmatrix}
$$

Secondary State Equations

$$\dot{X}_2 = \underline{0}$$

Measurement Equations

$$\beta 1_m = k_{\beta 1} \left(\beta_o - \beta_{1_c} \cos(\psi_R + \phi_1) - \beta_{1_s} \sin(\psi_R + \phi_1)\right) + b_{\beta_1} + n_1$$

$$\vdots$$

$$\beta i_m = k_{\beta_i} \left(\beta_o - \beta_{1_c} \cos(\psi_R + \phi_i) - \beta_{1_s} \sin(\psi_R + \phi_i)\right) + b_{\beta_i} + n_i$$

$$\vdots$$

$$\beta 6_m = k_{\beta_6} \left(\beta_o - \beta_{1_c} \cos(\psi_R + \phi 6) - \beta_{1_s} \sin(\psi_R + \phi 6)\right) + b_{R_6} + n_6$$

$$\zeta 1_m = k_{\zeta_1} \left(\zeta_o - \zeta 1_c \cos(\psi_R + \theta_1) - \zeta 1_s \sin(\psi_R + \theta_1)\right) + b_{\zeta_1} + n_7$$

$$\vdots$$

$$\zeta_{im} = k\zeta_i \left(\zeta_o - \zeta_{1_c} \cos(\psi_R + \theta_i) - \zeta 1_s \sin(\psi_R + \theta_i)\right) + b_{\zeta i} + n_{i+6}$$

$$\vdots$$

$$\zeta_{6m} = k_{\zeta 6} \left(\zeta_o - \zeta_{1_c} \cos(\psi_R + \theta_6) - \zeta 1_s \sin(\psi_R + \theta_6)\right) + b_{\zeta_6} + n_{12}$$

$$\cos\psi R_m = k_{\cos_{\psi R}} \cdot \cos(\psi_R) + b_{\cos_{\psi R}} + n_{13}$$

The parameters used in these equations are defined in the rotor state estimator parameter list shown in Table 2.4.

2.3.3 RSRA Estimator Equations

The uncompressed state and measurement vectors for the RSRA estimator are shown in Table 2.5. As before, the user has the option to flag the desired states, measurements and process noise sources; but, in this case it is not advisable to do so.

Table 2.4

Rotor State Estimator—Parameter List

PARAMETER INDEX	PARAMETER DESCRIPTION	
1-14	$\underline{x}_1$ ic's (uncompressed vector)	
15-40	$\underline{x}_2$ ic's (uncompressed vector)	
41-76	not used	
77	ϕ_1	Blade flapping measurement phase angles, rad.
78	ϕ_2	
79	ϕ_3	
80	ϕ_4	
81	ϕ_5	
82	ϕ_6	
83	θ_1	Blade lagging measurement phase angles, rad.
84	θ_2	
85	θ_3	
86	θ_4	
87	θ_5	
88	θ_6	

Table 2.5

RSRA Estimator-Vector Definitions (Uncompressed)

$$\underset{10\times1}{\underline{x}_1} = \begin{bmatrix} X_R \\ Y_R \\ Z_R \\ L_R \\ M_R \\ Q_R \\ a_{t_x} \\ a_{t_y} \\ a_{t_z} \\ Q_T \end{bmatrix} \qquad \underset{20\times1}{\underline{x}_2} = \begin{bmatrix} b_A \\ b_B \\ b_C \\ b_D \\ b_E \\ b_F \\ b_{at_x} \\ b_{at_y} \\ b_{at_z} \\ b_{QT} \\ k_A \\ k_B \\ k_C \\ k_D \\ k_E \\ k_F \\ k_{at_x} \\ k_{at_y} \\ k_{at_z} \\ k_{QT} \end{bmatrix} \qquad \underset{10\times1}{\underline{y}} = \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \\ a_{tx_m} \\ a_{ty_m} \\ a_{tz_m} \\ Q_{t_m} \end{bmatrix} \begin{matrix} \text{lbs.} \\ \text{lbs.} \\ \text{lbs.} \\ \text{lbs.} \\ \text{lbs.} \\ [illegible] \\ [illegible]^2 \\ [illegible]^2 \\ [illegible] \\ \text{ft-lb} \end{matrix}$$

$$\underset{10\times1}{\underline{w}} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \\ w_9 \\ w_{10} \end{bmatrix} \qquad \underset{6\times1}{\underline{u}} = \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \\ p \\ q \\ r \end{bmatrix} \begin{matrix} \text{rad/sec}^2 \\ \text{rad/sec}^2 \\ \text{rad/sec}^2 \\ \text{rad/sec} \\ \text{rad/sec} \\ \text{rad/sec} \end{matrix}$$

The integrity of the RSRA estimator requires that all measurements be available.

The $\underline{x}_1$ vector is a 10x1 vector of primary states for the RSRA estimator. These states are: the rotor forces at the hub in the aircraft body axis system - X_R, Y_R, Z_R; the rotor moments at the hub - L_R, M_R, N_R; the inertial accelerations of the transmission in the body axis system - a_{t_x}, a_{t_y}, a_{t_z}; and the sum of the engine and tail rotor drive shaft torques - Q_t.

The $\underline{y}$ vector is a 10x1 vector of measurements consisting of: the transmission load cell reactive forces - A,B,C,D,E,F: transmission accelerations - $a_{t_{x_m}}, a_{t_{y_m}}, a_{t_{z_m}}$; and total drive shaft torques - Q_t.

The $\underline{x}_2$ vector is a 20x1 vector of biases and scale factors for each of the 10 measurements.

The $\underline{u}$ vector is a 6x1 control vector of body angular accelerations - $\dot{p},\dot{q},\dot{r}$ and body angular rates -p,q,r that are treated as deterministic inputs. These six quantities should be obtained from a prior fuselage/gust estimator run.

The $\underline{w}$ vector is a 10x1 vector of process noise sources that drives each of the states.

The state and measurement equations for the RSRA estimator are presented below:

Primary State Equations

$$\dot{\underline{x}}_1 = \underline{w}$$

Secondary State Equations

$$\dot{\underline{x}}_2 = 0$$

Measurement Equations (bias & scale factors not shown)

$$A = h_{11}X_R + h_{12}Y_R + H_{13}Z_R + h_{14}L_R + h_{15}M_R + h_{16}Q_R - h_{11}M_t a_{t_x}$$

$$- h_{12}m_t a_{t_y} - h_{13}m_t a_{t_z} - h_{14}I_{t_{xx}}\dot{p} - h_{14}(I_{t_{zz}} - I_{t_{yy}})\, r\, p$$

$$+ h_{14}f_6 m_t a_{t_y} - h_{14}Q_t - h_{15}I_{t_{rr}}\dot{q} - h_{15}(I_{t_{xx}} - I_{t_{zz}})\, q\, r$$

$$+ h_{15}f_5 M_t a_{t_z} - h_{15}f_6 m_t a_{t_x} - h_{16}I_{t_{zz}}\dot{r} - h_{16}(I_{t_{yy}} - I_{t_{xx}})\, pq$$

$$- h_{16}f_5 m_t a_{t_y} + n_1$$

$$B = h_{21}X_R + h_{22}Y_R + h_{23}Z_R + h_{24}L_R + h_{25}M_R + h_{26}Q_R - h_{21}M_t a_{t_x}$$

$$- h_{22}M_t a_{t_y} - h_{23}M_t a_{t_z} - h_{24}I_{t_{xx}}\dot{p} - h_{24}(I_{t_{zz}} - I_{t_{yy}})\, r\, p$$

$$+ h_{24}f_6 M_t a_{t_y} - h_{24}Q_t - h_{25}I_{t_{yy}}\dot{q} - h_{25}(I_{t_{xx}} - I_{t_{zz}})\, q\, r$$

$$+ H_{25}f_5 m_t a_{t_z} - h_{25}f_6 m_t a_{t_x} - h_{26}I_{t_{zz}}\dot{r} - h_{26}(I_{t_{yy}} - I_{t_{xx}})\, p\, q$$

$$- h_{26}f_5 M_t a_{t_y} + h_2$$

$$C = \frac{f_3}{(f1+f2)} X_R - \frac{f_1}{(f1+f2)} Z_R - \frac{1}{(f1+f2)} M_R - 1 \frac{f_3}{(f1+f2)} m_t a_{t_x}$$

$$+ \frac{f_1}{(f1+f2)} m_t a_{t_z} + \frac{I_{t_{yy}}}{(f1\_f2} \dot{q} + \left(\frac{I_{t_{xx}} - I_{t_{zz}}}{(f1+f2)}\right) qr - \frac{f_5 m_t}{(f1+f2)}$$

$$a_{t_z} + \frac{f_6 m_t}{(f1+f2)} a_{t_x} + n_3$$

$$D = \frac{f_2}{(f1+f3)} Y_R - \frac{1}{(f1+f3)} Q_R + \frac{f_2}{(f1+f3)} m_t a_{t_y} + \frac{I_{t_{zz}}}{(f1+f3)}$$

$$+ \frac{(I_{t_{yy}} - I_{t_{xx}})}{(f1+f3)} pq + \frac{f_5 m_t}{(f1+f3)} a_{t_y} + n_4$$

$$E = \frac{f1+f3-f2}{f1+f3} Y_R - \frac{1}{(f1+f3)} Q_R - \frac{f1+f3-f2}{f1+f3} m_t a_{t_y} = \frac{I_{t_{zz}}}{(f1+f3)} \dot{r}$$

$$+ \frac{(I_{t_{yy}} - I_{t_{xx}})}{(f1+f3)} pq + \frac{f5\, m_t}{(f1+f3)} a_{t_y} + n_5$$

$$F = X_R - m_t a_{t_x} + n_6$$

$$a_{t_{x_m}} = a_{t_x} + n_7$$

$$a_{t_{y_m}} = a_{t_y} + n_8$$

$$a_{t_{z_m}} = a_{t_z} + n_9$$

$$Q_{t_m} = Q_t + n_{10}$$

where,

$$h_{11} = h_{21} = -\frac{1}{2} \frac{f_3}{(f1+f2)}$$

$$h_{12} = -h_{22} = -\frac{f_3}{V_t} + \frac{(f_4 - f_3) f_2}{V_t (f1+f3)}$$

$$h_{13} = h_{23} = -\frac{1}{2}\,\frac{f_2}{(f1+f2)}$$

$$h_{14} = -h_{24} = \frac{1}{y_t}$$

$$h_{15} = h_{25} = \frac{1}{2(f1+f2)}$$

$$h_{16} = -h_{26} = \frac{(f4-f3)}{y_t(f1+f3)}$$

The parameters used in these equations are defined in Table 2.6 (RSRA State Estimator - Parameter List) and in Figure 2.3.

Table 2.6

RSRA State Estimator – Parameter List

PARAMETER INDEX	PARAMETER DESCRIPTION	
1-10	$\underline{x}$, ic's (uncompressed vector)	
11-30	$\underline{x}_z$ ic's (uncompressed vector)	
31	y_t	
32	f_1	
33	f_2	
34	f_3	Defined below
35	f_4	
36	f_5	
37	f_6	
38	m_t	Transmission mass, slugs
39	I_{txx}	Transmission principle moments of inertia, slug-ft^2
40	I_{tyy}	
41	I_{tzz}	

where,

$$f_1 = \frac{x_t}{2} \cos i_t + (z_R + z_t) \sin i_t$$

$$f_2 = \frac{x_t}{2} \cos i_t - (z_R + z_t) \sin i_t$$

$$f_3 = (z_R + z_t) \cos i_t + \frac{x_t}{2} \sin i_t$$

$$f_4 = (z_R + z_t) \cos i_t - \frac{x_t}{2} \sin i_t$$

$$f_5 = z_R \sin i_t$$

$$f_6 = z_R \cos i_t$$

Table 2.6 (Concluded)

and,	
x_t, y_t, z_t	– are the rotor mounting geometry dimensions in the transmission principal axis system, ft.
z_R	– is the distance from the rotor hub to the transmission center of gravity along the shaft ft.
i_t	– transmission incidence with respect to the longitudinal body axis.

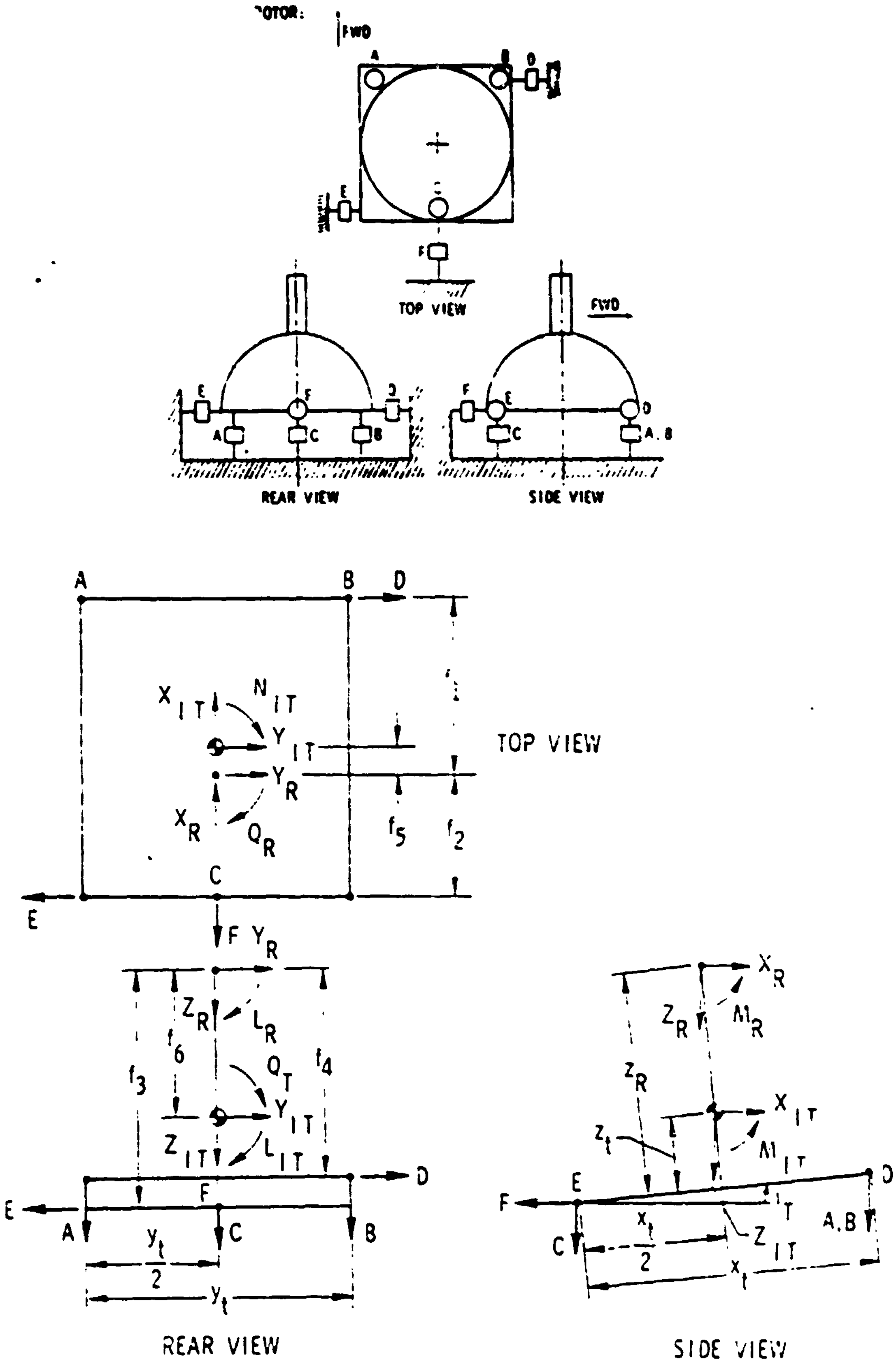


Figure 2.3 RSRA Rotor Hub Force/Moment Measurement System Configuration

III. PROGRAM DESCRIPTION

Two forms of input are used by the DEKFIS program. Information about the program options, parameter values, initial conditions and covariances are read from cards. The time histories of the measurements and controls are read from a mass storage device such as magnetic tape or disk.

3.1 CARD INPUT

All data cards are read by subroutine INPUT. The DEKFIS input card deck setup is discussed in Table 3.1. This table lists the input cards in order according to card type, indicating the numbers of each type of card required. Format and variable descriptions are discussed for each card type.

3.2 EXAMPLE INPUT DECK

An example input deck is shown in Figure 3.1. This deck is setup to process CH-53A flight data through the fuselage/gust state estimator. It is to be run in the (standard) combined primary and secondary state filter mode without any locally iterated smoothing. The output for this deck setup is discussed in Chapter IV.

3.3 TAPE AND DISK FILE REQUIREMENTS

The tape and disk file requirements are shown in Table 3.2. There is one input file which is required for <u>any</u> filter run. This input file contains the measurement ($\underline{y}_m$) and control ($\underline{u}$) time histories. There are two output files which are optional. These contain state, measurement and process noise estimates from a filter or fixed interval smoother run. A define file is required for any fixed interval smoother (FIS) run. This file is

Table 3.1
DEKFIS Input Card Setup

CARD TYPE	NUMBER OF TYPE CARDS	COLS.	FORMAT	NAME	DESCRIPTION
1	1	1-78	13A6	TITLE	Alphanumeric title for problem
2	1	1-5	I5	MODEL	1 for Fuselage/Gust State Estimator 2 for Rotor State Estimator 3 for RSRA State Estimator
		6-10	I5	MFILTR	If ≠ 0 forward filter is executed 1 for primary state (bias free) filter only 2 for combined primary and secondary state filter (with bias calculations) 3 for combined filter followed by bias free filter (preliminary to FIS run)
		11-15	I5	MSMOTH	If ≠ 0 the fixed interval smoother (FIS) will be executed
		16-20	I5	MBFLS	Number of bias free local smooths in a forward filter iteration
		21-25	I5	MCSLS	Number of composite state local smooths in a forward filter iteration
		26-30	I5	MDEFIL	>0 sets up define file for storage of matrices for FIS
3	1	1-5	I5	MDP	Number of data points (sample steps) - includes t=0 point
		6-10	I5	MX	Number of primary states (compressed vector length)
		11-15	I5	MX2	Number of secondary states (compressed vector length) (≥ 1)
		16-20	I5	MU	Number of controls (≥ 1)
		21-25	I5	MP	Number of process noise sources (≥ 1)
		26-30	I5	MY	Number of measurements
		31-35	I5	MPARAM	Parameter vector length (use 105)
		36-40	I5	MBLOCK	Not used in this version

Table 3.1 (Continued)

CARD TYPE	NUMBER OF TYPE CARDS	COLS.	FORMAT	NAME	DESCRIPTION
4	1	1-10	F10.0	DT	Time interval between sample points of the time history data
5	1	1-5	I5	INTH	= +k, then the first k sample points of the measurement and control time histories read into the program will be printed = -k, then every kth sample point will be printed = 0, if they are not to be printed
		6-10	I5	LVLPRT	Sets level of detail of the diagnostic printout: =3, highest level - very detailed =2, $\hat{x}$, $\hat{b}$, P_x, P_b =1, minimal
		11-15	I5	INCPR2	Number of data points between diagnostic printout at level 2 (Active if LVLPRT $\geq$ 2)
		16-20	I5	INCPR3	Number of data points between diagnostic printout at level 3. (Note: at level 3 the diagnostic printout for the first three points is always printed, regardless of INCPR3 value)
		21-25	I5	JDUMPD	>0, then print D array as part of level 3 diagnostic printout. (D array contains additional diagnostic information -- see text).
		26-30	I5	IDUM(1)	If >0, then sets logical unit of input file containing measurement and control time histories (Default is 2)
		31-35	I5	IDUM(2)	If >0, then sets logical unit of output file to which time, measurements and measurement estimates will be written (Default is 3)
		36-40	I5	IDUM(3)	If >0, then sets logical unit of output file to which the estimated state time histories will be written (Default is 4)
		41-45	I5	IDUM(4)	If >0, then sets logical unit of define file used to store matrices needed by the smoother (Default is 8)

Table 3.1 (Continued)

CARD TYPE	NUMBER OF TYPE CARDS	COLS.	FORMAT	NAME	DESCRIPTION
6	1	1-80	80I1	IDUM(J), J=1,MXMX2T	Flags elements of the total (uncompressed) primary + secondary state vector: (See Tables 2.1, 2.3 or 2.5) =0, not used =1, used =2, used as control only The values greater than zero are used to set NSFLAG and ISFLAG vectors to compress the state vector and associated matrices for computation.
7	1	1-80	80I1	IDUM(J), J=1,MYT	Flags elements of the total (uncompressed) measurement vector for usage: (See Tables 2.1, 2.3 or 2.5) = 0, not used $\geq$ 1, used Sets NMFLAG vector for subsequent compression of the measurement vector.
8	1	1-80	80I1	IDUM(J), J=1,MPT	Flags elements of the total (uncompressed) process noise vector for usage: (See Tables 2.1, 2.3 or 2.5) = 0, not used $\geq$ 1, used Sets NPFLAG vector for subsequent compression.
9	1	1-5	I5	ISTORX	Rate of storage of $\hat{x}_1$ vector (used by output subroutines called after all computation has been completed)
		6-10	I5	ISTORB	Rate of storage of $\hat{x}_2$ vector (stored for forward filter only - not stored for fixed interval smoother)
		11-15	I5	ISTORW	Rate of storage of $\hat{w}$ vector (stored for smoother only)

Table 3.1 (Continued)

CARD TYPE	NUMBER OF TYPE CARDS	COLS.	FORMAT	NAME	DESCRIPTION
9		16-20	I5	ISTORY	Rate of storage of y_m and $\hat{y}$ vectors
		21-25	I5	ISTORA	Rate of storage of $\hat{\ddot{x}}$, acceleration vector (stored for forward filter only). If the rate of storage is: =1, every data point is stored =2, every other point is stored ⋮ =n, every nth point is stored.
10	3				First card is for the forward filter in the combined mode (L=1). Second card is for the forward filter in the bias free mode (L=2). Third card is for the fixed interval smoother pass (L=3). It is possible to execute this program so that the forward filter runs in the combined primary and secondary state mode followed by a bias free filter pass and a third fixed interval smoother pass (i.e., MFILTR=3, MSMOTH=1. In this case, it might be desirable to look at output from each pass).
		1-75	15I5	((IBNXY(J,K,L), J=1,5), K=1,3)	In three output mode groups w/ five output flags each: Output Mode Group: K=1, Tape or file storage $\hat{x}_1$, $\hat{\dot{x}}_1$, $\hat{x}_2$, $\hat{w}$ written to disc file 4 (or as set on card 5). Each is separated by an end of file mark. t, y_m, y are written to tape file 3 (or as set on card 5). K=2, Print K=3, Printer Plots

Table 3.1 (Continued)

CARD TYPE	NUMBER OF TYPE CARDS	COLS.	FORMAT	NAME	DESCRIPTION
10 (Cnt'd)					<u>Output Flags</u> J=1, Sets flag to output the primary states $\hat{x}_1$ in the specified output mode. J=2, Sets flag to output the secondary states $\hat{x}_2$ in the specified output mode. J=3, Sets flag to output the process noise estimates $\hat{w}$ in the specified output mode. J=4, Sets flag to output the measurements y_m and the measurement estimates $\hat{y}$ in the specified mode. J=5, Sets flag to output the primary state accelerations $\hat{\ddot{x}}_1$. Available for file storage (K = 1) only. The value of IBWXY indicates the frequency of output relative to what has been stored (see card 9). Thus, the actual rate of output for the primary states is the product of ISTORX and IBWXY(1,K,L).
11	Maximum 106. Use one card for each nonzero parameter. Last card has * only.	1	A1	ECHK	= blank, if this card contains parameter information = *, if no more cards of this type are to be read.
		2-5	I4	J	Parameter index (See Tables 2.2, 2.4 or 2.6)
		6-25	F20.0	P	Parameter value
12	Maximum MX*MX+1. One card for each nonzero initial state covariance matrix element.	1	A1	ECHK	= blank, if this card contains initial state covariance matrix - $P_{x_1}(0)$ information (symmetric matrix) = *, if no more cards of this type are to be read
		2-5	I4	J	Row index
		6-10	I5	K	Column index
		11-30	F20.0	A	Value of matrix element

Table 3.1 (Concluded)

CARD TYPE	NUMBER OF TYPE CARDS	COLS.	FORMAT	NAME	DESCRIPTION
13	Maximum MX2*MX2 +1. One card for each non-zero initial bias covariance matrix element.	1 2-5 6-10 11-30	A1 I4 I5 F20.0	ECHK J K A	Refer to card type 12. This set of cards defines the initial bias (secondary states x_2 covariance matrix - $P_{x_2}(0)$ (symmetric matrix).
14	Maximum MP*MP+1. One card for each nonzero process noise covariance element.	1 2-5 6-10 11-30	A1 I4 I5 F20.0	ECHK J K A	Refer to card type 12. This set of cards defines the process noise covariance matrix, Q (symmetric matrix).
15	Maximum MY*MY+1. One card for each nonzero measurement noise covariance element.	1 2-5 6-10 11-30	A1 I4 I5 F20.0	ECHK J K A	Refer to card type 12. This set of cards defines the measurement noise covariance, R (symmetric matrix).
16	One if $M \leq 15$. Two if $M > 15$.	1-5 6-	I5 MI5	NITEMS IDUM(J), J=1,M	Number of data channels on the input tape (default unit 2) for a given time point M=MY+MU, the total number of measurements + controls needed by the program. IDUM is a sorting vector indicating what tape channel is used for the appropriate measurement or control, i.e., IDUM(3) = 25 indicates the 25th channel on the input tape is used for the third measurement.

```
                     DATA CARDS                                              CARD TYPE
1    FUSELAGE GUST STATE ESTIMATOR    CH-53A FLIGHT DATA                        1
    1    2    0    0    0   -1                                                  2
  550   14    9    1    6   14  105    0                                        3
0.05                                                                            4
   10    7  100 1000    0    2    3    4    8                                   5
11 1111111111111         11             1     1   11       111                  6
11    111111111    111                                                          7
111111                                                                          8
    1    1    0    1    1                                                       9
    1    0    0    1    1    0    0    0    0    0    1    1    0    1    0    } 10
    1    0    0    1    1    0    0    0    0    0    1    1    0    1    0    } BLANK CARD
    1   0.00054                                                               ┐
    2   0.04650                                                               │
    4    162.5                                                                │
    5    -8.11                                                                │
    6    -5.90                                                                │
    7    -0.0140                                                              │
    8    -0.0063                                                              │
    9    0.0005                                                               │
   10    1.069                                                                │
   11    -.3294                                                               │
   12    1.292                                                                │
   13    0.30                                                                 │
   14    -0.38                                                                │
   15    0.38                                                                 │
   26    0.0                                                                  } 11
   27    0.011                                                                │
   28    -7.8889                                                              │
   32    0.0                                                                  │
   33    -0.0013                                                              │
   34    0.0                                                                  │
   35    0.73                                                                 │
   37    -.25                                                                 │
   36    -.11                                                                 │
   38    -.0005                                                               │
   39    -0.0095                                                              │
   40    0.0                                                                  │
   44    0.0                                                                  │
   45    0.0                                                                  │
   46    -0.300                                                               │
   50    0.44                                                                 │
   51    0.71                                                                 ┘
```

Figure 3.1 Sample Input Listing

DATA CARDS			CARD TYPE
52	1.0		11
56	1.0		
57	1.0		
58	1 0		
59	1.0		
60	1.0		
61	-1.0		
62	1.0		
63	1.0		
64	-1.0		
68	1.0		
69	1.0		
70	1.0		
76	-0.15		
77	0.0		
78	-0.725		
79	-0.15		
80	0.0		
81	-0.725		
82	-0.15		
83	0.0		
84	-0.725		
85	31.600		
86	4.083		
87	4.900		
88	31.392		
89	4.083		
90	3.942		
91	31.392		
92	5.083		
93	4.400		
94	-10.0		
95	-10.0		
96	-10.0		
97	-10.0		
98	-10.0		
99	-10.0		
1	1	0.000002	
2	2	0.000002	
3	3	2.25	
4	4	2.25	
5	5	2.25	
6	6	0.0001	

Figure 3.1 (Continued)

DATA CARDS			CARD TYPE
7	7	0.0001	12
8	8	0.0001	
9	9	0.025	
10	10	0.025	
11	11	0.10	
12	12	0.005	
13	13	0.005	
14	14	0.005	
•			
1	1	0.0001	13
2	2	0.0001	
3	3	0.00001	
4	4	0.0	
5	5	0.001	
6	6	0.001	
7	7	0.001	
8	8	0.001	
9	9	0.001	
•			
1	1	0.8	14
2	2	0.8	
3	3	0.8	
4	4	0.02	
5	5	0.02	
6	6	0.02	
•			
1	1	0.0001	15
2	2	0.0001	
3	3	0.000001	
4	4	0.000001	
5	5	0.000001	
6	6	0.01	
7	7	0.1	
8	8	0.1	
9	9	0.001	
10	10	0.001	
11	11	0.001	
12	12	2.25	
13	13	0.0001	
14	14	0.0001	
•			

37 1 2 4 5 6 11 10 12 7 8 9 13 15 14 16

Figure 3.1 (Concluded)

Table 3.2

Tape and Disk File Requirements

FILE TYPE	DEFAULT LOGICAL UNIT	DESCRIPTION	COMMENTS
INPUT	2	• Necessary for any filter run • Contains measurement $\underline{y}_m$ and control $\underline{u}$ time histories	• Usually flight data • Written in binary • Read in by INREAD
OUTPUT (#1)	3	• Optional output • Contains time t, measurements $\underline{y}_m$, and measurement estimates $\hat{\underline{y}}$	
OUTPUT (#2)	4	• Optional output • Contains up to 4 vector time histories • Choice of primary state estimates $\hat{x}_1$, accelerations (prediction cycle) $\dot{\hat{x}}_1$, secondary state estimates $\hat{x}_2$ and process noise $\hat{w}$.	• Written by WRTDAT • WRTDAT is called once for each vector to be stored and an EOF written after each
DEFINE FILE	8	• Used to store matrices from filter runs for use in the fixed interval smoother runs • Used as input to FIS run	• File read/write compatibility handled automatically

used to store matrixes at every time point in the forward filter pass for use in the subsequent backward FIS run.

Prior to the initial DEKFIS runs, an input file must be set up on mass storage. This file must be compatible with the FORTRAN code in the INREAD subroutine. This input file is to be written in binary with one record written for each time point. Each record contains the measurement and control values at the given time point. The FORTRAN code below illustrates this further:

```
      DIMENSION  RECORD (NITEMS)
      DO  10  J = 1, MDP
      WRITE(2)  RECORD
   10 CONTINUE
```

where,

MDP = number of data (or time) points

NITEMS = number of measurement and control channels in each record

RECORD = an array of length NITEMS containing the measurement and control values needed by the DEKFIS run

Note: NITEMS can be greater than actually needed for the run. When running DEKFIS, card type 16 tells INREAD the value of NITEMS and which of these NITEMS channels are actually to be used. Rearranging is also possible with card type 16.

3.4 JOB SUBMISSION ON CDC-7600 MACHINES

The DEKFIS program was developed on a CDC-7600 and requires 103777 (octal) small core memory (SCM) and 141520 (octal) large core memory (LCM).

The JCL (job control language) for a typical DEKFIS run is shown in Figure 3.2. This figure shows an update, compile, load and go sequence. The output files are cataloged after a successful execution.

```
                        SCOPE 2.1.3 205-210 03/02/80 1 03/17/80       M0077

 SYS. DEVICES 817/5P7    844/SAC/111/    FLS=0200K FLL=1750K ARS=0160K MXL=1200K MXB=12678

HH.MM.SS CPU SECOND ORIGIN
         11.07.06.ARS. XP        AMES  SCOPE 3.4.4    414-9     03/04/79
11.35.33 00000.003 ARC.      -SOE,T150,Y01.
11.35.33 00000.003 JOB.      -ACCOUNT
11.35.43 00000.111 JOB.      -MOUNT,VSN=D0185A,SN=FSOMOHA.
         11.35.43 ARC.          JM274 - JOB ROLLED - REQUESTING A VD
11.37.34 00000.134 ARC.         RP570 - VSN D0185A OF SET FSOMOHA MOUNTED
11.37.34 00000.135 JOB.      -SETNAME=FSOMOHA.
11.37.34 00000.135 JOB.      -REQUEST,TAPE4,*PF,SN=FSOMOHA.
11.37.37 00000.138 JOB.      -REQUEST,TAPE1,*PF,SN=FSOMOHA.
11.37.37 00000.143 JOB.      -ATTACH,TAPE2,SIS3TAP,ID=RSH,SN=FSOMOHA.
11.37.38 00000.147 ARC.         PF255 - CYCLE     2 ATTACHED FROM SN=FSOMOHA
11.37.38 00000.148 JOB.      -ATTACH,OLDPL,FILTER,ID=BJR.
11.37.38 00000.151 ARC.         PF255 - CYCLE     1 ATTACHED FROM SN=FSOMOHA
11.37.38 00000.152 JOB.      -UPDATE.
11.37.42 00000.157 JSP.       READING INPUT
11.37.42 00000.173 ARC.         RP727 - VSN D0185A OF SET FSOMOHA MOUNTED
11.37.48 00000.532 JSR.       UPDATE COMPLETED
11.37.48 00000.533 JOB.      -RETURN,OLDPL.
11.37.50 00000.541 JOB.      -FTN(4,R=3,OPT=1,I=COMPILE)
11.37.56 00000.557 JSR.         NULL PROGRAM IGNORED AFTER  *B-D-I*
11.37.59 00000.574 JSR.               .030 CP SECONDS COMPILATION TIME
11.37.59 00000.575 JOB.      -RETURN(COMPILE)
11.38.02 00000.585 JOB.      -ATTACH,FILTRN,FILTERRN,ID=SBD.
11.38.02 00000.587 ARC.         PF255 - CYCLE     1 ATTACHED FROM SN=FSOMOHA
11.38.02 00000.587 JOB.      -LOAD,FILTRN.
11.38.02 00000.590 JOB.      -EXECUTE.
11.38.03 00000.600 ARC.         RP727 - VSN D0185A OF SET FSOMOHA MOUNTED
11.38.30 00005.010 ARC.         LO610 - FLS REQUIRED TO LOAD - 0023235 OU.CUG
11.38.30 00005.012 ARC.         LO603 - EXECUTION INITIATED 05.EXP
11.38.30 00005.012 JSR.       FORTRAN LIBRARY 460-3          05/01/79
11.38.33 00005.124 ARC.         RP727 - VSN D0185A OF SET FSOMOHA MOUNTED
11.41.32 00077.945 ARC.         RP727 - VSN D0185A OF SET FSOMOHA MOUNTED
11.41.33 00077.195 ARC.         RP727 - VSN D0185A OF SET FSOMOHA MOUNTED
```

Figure 3.2 JCL for a Typical DEKFIS Run

IV. PROGRAM OUTPUT

Most of the program's output is written by the printer, including optional pointer plots. The program also has the option of writing data on mass storage devices (see Figure 3.2), which gives the user the information necessary to make off-line, pen-and-ink plots (e.g., Calcomp plots).

4.1 PRINTED OUTPUT

Examples of program output are shown in Figures 4.1 to 4.7. These are explained below:

Figure 4.1 Contains the program title block and a summary of the major inputs specified via card input. These include: the mode of operation, model type and size, sample step size and number, diagnostic printout flags, and variable selection flag vectors.

Figure 4.2 Contains listings of the primary states, secondary states, process noises and measurements used in the run. A summary table of the output increments is also printed out. The remaining information is used primarily for diagnostic purposes.

Figure 4.3 Shows the parameter vector as input on card type 11, the initial covariance of the primary states (PS) as input on card type 12, and the initial covariance of the secondary states (PB) as input on card type B.

Figure 4.4 Shows the covariances of the process noise and measurement noise as input by card types 14 and 15. It also contains printout generated by INREAD as it reads the input data file into local storage.

```
1    FUSELAGE GUST STATE  ESTIMATOR   CH-53A FLIGHT DATA                                              *DEKFIS PROGRAM*

*****************************************************************************************************
*                                                                                                   *
*                                          D E K F I S                                              *
*                                          -----------                                              *
*                                                                                                   *
*                           DISCRETE EXTENDED KALMAN FILTER/SMOOTHER PROGRAM                        *
*                                                                                                   *
*                                          DEVELOPED BY                                             *
*                                     SYSTEMS CONTROL, INC. (VT.)                                   *
*                                        1801 PAGE MILL ROAD                                        *
*                                       PALO ALTO, CAL. 94304                                       *
*                                                                                                   *
*****************************************************************************************************

MODE OF OPERATION:
     THE FORWARD FILTER TO ESTIMATE BOTH PRIMARY AND SECONDARY STATES WILL BE EXECUTED.
          NUMBER OF BIAS-FREE STATE LOCAL SMOOTH ITERATIONS PER STEP =  0
          NUMBER OF COMPOSITE-STATE LOCAL SMOOTH ITERATIONS PER STEP =  0

MODEL:
     FUSELAGE/GUST EQUATIONS              (NO. 1)

     PRIMARY STATES. . . . . . . . . .    14
     SECONDARY STATES. . . . . . . . .     9
     CONTROLS. . . . . . . . . . . . .     1
     PROCESS NOISE SOURCES . . . . . .     6
     MEASUREMENTS. . . . . . . . . . .    14
     PARAMETERS. . . . . . . . . . . .   105

SAMPLE STEP:
     NUMBER OF SAMPLE STEPS. . . . . .   550
     STEP-SIZE . . . . . . . . . . . .  .050

DIAGNOSTIC PRINT:
     LEVEL . . . . . . . . . . . . . .     2
     INCREMENT AT LEVEL 2. . . . . . .   100
     INCREMENT AT LEVEL 3. . . . . . .  1000

LOGICAL UNITS:
     INTAPE =    2.  INPUT FILE CONTAINING MEASUREMENT AND CONTROL TIME HISTORIES.
     OUTAPE =    3.  OUTPUT FILE CONTAINING TIME, MEASUREMENT, AND ESTIMATED MEASUREMENT TIME HISTORIES.
     OUTFIL =    4.  OUTPUT FILE CONTAINING ESTIMATED STATE TIME HISTORIES.
     DEFILL =    8.  DEFINE FILE UNIT FOR STORING MATRICES NEEDED BY THE SMOOTHER.

VARIABLES SELECTED:

     NSFLAG =   1  2  4  5  6  7  8  9 10 11 12 13 14 15  1  2 15 21 25 26 34 35 36
     ISFLAG =   1  2  0  1  4  5  6  7  8  9 10 11 12 13 14  0  0  0  0  0  0  0  0  0  0 15 16  0  0  0  0  0  0  0  0
                0  3  0  0 17  0  0  0  0  0 18  0  0  0 19 20  0  0  0  0  0  0  0 21 22 23  0  0  0  0  0  0  0  0  0
                0  0  0  0  0
     NMFLAG =   1  2  7  3  9 10 11 12 13 14 15 19 20 21
     NPFLAG =   1  2  3  4  5  6
```

Figure 4.1 Title Block and Summary

PRIMARY STATES	SECONDARY STATES	PROCESS NOISES	MEASUREMENTS
PHI	PHI BIAS	AXI NOISE	PHI MEAS
THETA	THETA BIAS	AYI NOISE	THETA MEAS
VX, LONG VEL	RDOT BIAS	AZI NOISE	P MEAS
VY, LAT VEL	ALPHA BIAS	PDOT NOISE	Q MEAS
VZ, VERT VEL	PHI S.F.	QDOT NOISE	R MEAS
P, ROLL RATE	THETA S.F.	RDOT NOISE	AXI MEAS
Q,PITCH RATE	AXI S.F.		AYI MEAS
R, YAW RATE	AYI S.F.		AZI MEAS
AXI, LONG AC	AZI S.F.		PDOT MEAS
AYI, LAT AC			QDOT MEAS
AZI, VERT AC			RDOT MEAS
PDOT			VEL MEAS
QDOT			BETA MAS
RDOT			ALPHA MEAS

OUTPUT INCREMENTS:

	FILTER PASS			
	STORE	TAPE	PRINT	PLOT
X, PRIMARY STATES	1	1	0	1
B, SECONDARY STATES	1	0	0	1
W, PROCESS NOISES	0	0	0	0
Y, MEASUREMENTS	1	1	0	1
XDOT, ACCELERATIONS	1	1	0	0

STORAGE LIMITS:
LAST USED LOCATION IN THE R ARRAY 39211
LAST AVAILABLE LOCATION IN THE R ARRAY. . 15000

*** SUBROUTINE ERROFF HAS DETECTED AN ERROR. NRTOTL = 15000 N = 39211
LAST USED LOCATION IN THE D ARRAY 5768
LAST AVAILABLE LOCATION IN THE D ARRAY. . 10000

MATRICES:

NAME	ROW	COL	START	NAME	ROW	COL	START	NAME	ROW	COL	START	NAME	ROW	COL	START
NXHAT	14	1	495	NXHATO	14	1	910	NXBAR	14	1	924	NY	14	1	938
NYHAT	14	1	952	NHHAT	9	1	966	NU	1	1	975	NF	14	14	976
NH	14	9	1177	NG	14	1	1298	NGAM	14	6	4690	NH	14	14	420
NHT	14	14	1312	NC	14	9	1508	ND	14	1	1634	NQ	6	6	1640
NR	14	1	1634	NRINV	14	1	1698	NPHI	14	14	616	NGD	14	1	1712
NDX	14	1	1726	NLINV	3	14	1740	NFLOT	3	14	1782	NEV	14	2	1824
NT	14	14	1852	NPS	14	14	2048	NKS	14	14	2244	NMS	14	14	2440
NUX	14	1	2636	NVX	14	9	2762	NPR	9	9	2888	NKB	9	14	2969
NS	14	9	3095	NZBIAS	9	1	3221	NPXB	14	9	3230	NNU	14	1	0
NMS	14	1	3356	NMHAT	14	1	3370	NXHATC	14	1	14	NPX	14	14	28
NX2	23	1	3384	NUHATO	7	1	3407	NPSO	14	14	3416	NXHATI	14	1	3612
NTLU	14	14	3626	NFINT	14	14	3822	NPSI	14	9	4018	NXBARO	14	1	4144
NPHU	9	1	4158	NXHCO	14	1	4739	NPXBD	9	9	4253	NPXO	14	14	4334
NVXO	14	1	4530	NKX	14	14	224	NWHAT	6	1	4656	NL	14	1	4662
NXHATS	14	1	4676	NGAMD	14	6	812								

DEFINE FILE:
A DEFINE FILE FOR THIS RUN REQUIRES 550 RECORDS OF 1792 WORDS EACH, WHICH EQUALS 551 TRACKS.

Figure 4.2 States, Measurements, and Noises Used

PARAMETER VALUES:

NO.	VALUE	NO.	VALUE	NO.	VALUE	NO.	VALUE	NO.	VALUE	NO.	VALUE
1	6.400000E-04	19	0.	37	-2.500000E-01	55	0.	73	0.	91	3.139200E+01
2	4.650000E-02	20	0.	38	-5.000000E-04	56	1.000000E+00	74	0.	92	5.083000E+00
3	0.	21	0.	39	-9.500000E-03	57	1.000000E+00	75	0.	93	4.900000E+00
4	1.625000E+02	22	0.	40	0.	58	1.000000E+00	76	-1.500000E-01	94	-1.000000E+01
5	-8.110000E+00	23	0.	41	0.	59	1.000000E+00	77	0.	95	-1.000000E+01
6	-5.700000E+00	24	0.	42	0.	60	1.000000E+00	78	-7.250000E-01	96	-1.000000E+01
7	-1.400000E-02	25	0.	43	0.	61	-1.000000E+00	79	-1.500000E-01	97	-1.000000E+01
8	-6.300000E-03	26	0.	44	0.	62	1.000000E+00	80	0.	98	-1.000000E+01
9	5.000000E-04	27	1.100000E-02	45	0.	63	1.000000E+00	81	-7.250000E-01	99	-1.000000E+01
10	1.069000E+00	28	-7.088930E+00	46	-3.000000E-01	64	-1.000000E+00	82	-1.500000E-01	100	0.
11	-3.294000E-01	29	0.	47	0.	65	0.	83	0.	101	0.
12	1.292000E+00	30	0.	48	0.	66	0.	84	-7.250000E-01	102	0.
13	3.000000E-01	31	0.	49	0.	67	0.	85	3.160000E+01	103	0.
14	-1.800000E-01	32	0.	50	4.400000E-01	68	1.000000E+00	86	4.083000E+00	104	0.
15	1.800000E-01	33	-1.300000E-03	51	7.100000E-01	69	1.000000E+00	87	4.900000E+00	105	0.
16	0.	34	0.	52	1.000000E+00	70	1.000000E+00	88	3.139200E+01		
17	0.	35	7.300000E-01	53	0.	71	0.	89	4.083000E+00		
18	0.	36	-1.100000E-01	54	0.	72	0.	90	3.942000E+00		

PS

	1	2	3	4	5	6	7	8	9	10
1	2.0000E-06	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	0.	2.0000E-06	0.	0.	0.	0.	0.	0.	0.	0.
3	0.	0.	2.2500E+00	0.	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	2.2500E+00	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	2.2500E+00	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	1.0000E-04	0.	0.	0.	0.
7	0.	0.	0.	0.	0.	0.	1.0000E-04	0.	0.	0.
8	0.	0.	0.	0.	0.	0.	0.	1.0000E-04	0.	0.
9	0.	0.	0.	0.	0.	0.	0.	0.	2.5000E-02	0.
10	0.	0.	0.	0.	0.	0.	0.	0.	0.	2.5000E-02
11	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
12	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
13	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
14	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

	11	12	13	14
1	0.	0.	0.	0.
2	0.	0.	0.	0.
3	0.	0.	0.	0.
4	0.	0.	0.	0.
5	0.	0.	0.	0.
6	0.	0.	0.	0.
7	0.	0.	0.	0.
8	0.	0.	0.	0.
9	0.	0.	0.	0.
10	0.	0.	0.	0.
11	1.0000E-03	0.	0.	0.
12	0.	5.0000E-03	0.	0.
13	0.	0.	5.0000E-03	0.
14	0.	0.	0.	5.0000E-03

Figure 4.3 Parameter Vector and Covariance

P9

	1	2	3	4	5	6	7	8	9
1	1.0000E-04	0.	0.	0.	0.	0.	0.	0.	0.
2	0.	1.0000E-04	0.	0.	0.	0.	0.	0.	0.
3	0.	0.	1.0000E-05	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	1.0000E-03	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	1.0000E-03	0.	0.	0.
7	0.	0.	0.	0.	0.	0.	1.0000E-03	0.	0.
8	0.	0.	0.	0.	0.	0.	0.	1.0000E-03	0.
9	0.	0.	0.	0.	0.	0.	0.	0.	1.0000E-03

Figure 4.3 (Continued)

Q

	1	2	3	4	5	6
1	8.0000E-01	0.	0.	0.	0.	0.
2	0.	8.0000E-01	0.	0.	0.	0.
3	0.	0.	8.0000E-01	0.	0.	0.
4	0.	0.	0.	2.0000E-02	0.	0.
5	0.	0.	0.	0.	2.0000E-02	0.
6	0.	0.	0.	0.	0.	2.0000E-02

R

	1	2	3	4	5	6	7	8	9	10
1	1.0000E-04	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	0.	1.0000E-04	0.	0.	0.	0.	0.	0.	0.	0.
3	0.	0.	1.0000E-06	0.	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	1.0000E-06	0.	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	1.0000E-06	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.	1.0000E-02	0.	0.	0.	0.
7	0.	0.	0.	0.	0.	0.	1.0000E-01	0.	0.	0.
8	0.	0.	0.	0.	0.	0.	0.	1.0000E-01	0.	0.
9	0.	0.	0.	0.	0.	0.	0.	0.	1.0000E-03	0.
10	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.0000E-03
11	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
12	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
13	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
14	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

	11	12	13	14
1	0.	0.	0.	0.
2	0.	0.	0.	0.
3	0.	0.	0.	0.
4	0.	0.	0.	0.
5	0.	0.	0.	0.
6	0.	0.	0.	0.
7	0.	0.	0.	0.
8	0.	0.	0.	0.
9	0.	0.	0.	0.
10	0.	0.	0.	0.
11	1.0000E-03	0.	0.	0.
12	0.	2.2500E+00	0.	0.
13	0.	0.	1.0000E-04	0.
14	0.	0.	0.	1.0000E-04

INPUT RECORDS:
NUMBER OF ITEMS IN THE TAPE RECORD. . . 17
ORDER = 1 2 4 5 6 11 10 12 7 8 9 13 15 14 0

*** NUMERICAL OVERFLOW DETECTED ON UNIT 2 ABS VALUE .GT. 1000000. SET EQUAL TO 0.0, IPOINT= 58

*** NUMERICAL OVERFLOW DETECTED ON UNIT 2 ABS VALUE .GT. 1000000. SET EQUAL TO 0.0, IPOINT= 176

Figure 4.4 Process Noise and Measurement Covariance

Figure 4.5 Contains printout from INREAD. The initial time points of the measurements and controls used in the run are printed out in order to allow verification that the correct data is being used.

Figure 4.6 Contains the intermediate printout of the performing filter. This is level 2 printout containing the state and bias estimates and the associated covariances at the indicated iteration. The level 3 printout (not shown) is extremely detailed giving all the intermediate steps in each iteration. It is <u>not</u> recommended that level 3 printout be used except for diagnostic purposes.

Figure 4.7 This figure shows a small portion of the printer plots generated by this run. The variables are in groups of four and are plotted at the rate of one line of printout for each time point as dictated by the print increment.

4.2 MASS STORAGE OUTPUT

The two types of mass storage output are shown in Figure 3.2. It is important to be able to read these files in order to set up runs for plotting, etc.

<u>Output File #1</u> - This file contains time, the measurements, and the measurement estimates. The program control logic for writing this file is conveyed to the DEKFIS run on card types 9 and 10. The file is written by subroutine OUTPUT, and the following FORTRAN code is supplied for reading the file:

```
      DIMENSION TITLE(18), YHAT(MY), YM(MY), UU(MU)
      READ(3) TITLE, MY, MU
      DO 10 J=1, NDPF
      READ(3) T, YHAT, YM, UU
   10 CONTINUE
```

where,
- TITLE – is the title information
- YHAT – is the $\hat{\underline{y}}$ vector of length MY
- YM – is the $\underline{y}_m$ vector of length MY
- UU – is the $\underline{u}$ vector of length MU
- MY – is the number of measurements
- MU – is the number of controls
- NDPF – is the number of data points on file
- T – is time

Output File #2 - This file can contain up to four vector time history files, written sequentially and separated by end-of-file marks. The user has the option of specifying which (if any) of the following four vector time histories are to be stored. These are (and in the order stored): the primary state estimates $\hat{x}_1$, the primary state accelerations (after the predict cycle of the Kalman filter iteration) $\hat{\ddot{x}}_1$, the secondary (or bias) state estimates and the process noise estimate $\hat{\underline{w}}$. The vector time histories are each written by WRTDAT as specified by the options of card type 10. An end-of-file mark is written by WRTDAT, so an EOF separates each section. The following FORTRAN is supplied for reading any one section:

```
      DIMENSION  TITLE(18), VECTOR(M)
      READ(4)  TITLE, M
      DO 10 J = 1, NDPF
      READ (4) K, VECTOR
   10 CONTINUE
```

1 FUSELAGE GUST STATE ESTIMATOR CH-53A FLIGHT DATA *DEKFIS PROGRAM*

MEASUREMENT TIME HISTORIES:

DATA PT.	TIME (SEC)	PHI MEAS (RAD)	THETA MEAS (RAD)	P MEAS (RAD/SEC)	Q MEAS (RAD/SEC)	R MEAS (RAD/SEC)	AXI MEAS (FT/SEC**2)	AYI MEAS (FT/SEC**2)	AZI MEAS (FT/SEC**2)
		PDOT MEAS (RAD/SEC**2)	QDOT MEAS (RAD/SEC**2)	RDOT MEAS (RAD/SEC**2)	VEL MEAS (FT/SEC)	BETA MAS (RAD)	ALPHA MEAS (RAD)		
0	0.0000	6.4891E-04	4.4698E-02	-1.3807E-02	-6.3355E-03	4.9393E-04	2.4900E+00	-3.2670E-01	3.085CE+01
		3.2044E-01	-1.7564E-01	3.7734E-01	1.6410E+02	-4.7996E-02	-3.3493E-01		
1	.0500	5.1250E-04	4.4418E-02	-9.6569E-03	-5.6514E-03	-1.3444E-03	2.5240E+00	-1.0470E-01	3.1270E+01
		-2.0492E-01	-6.9848E-02	-1.8623E-01	1.6230E+02	-4.5780E-02	-3.3720E-01		
2	.1000	6.4991E-04	4.4698E-02	-7.2344E-03	-4.2830E-03	-2.8152E-03	2.6160E+00	-1.1230E-02	3.1540E+01
		3.7088E-01	-3.8380E-01	5.8713E-01	1.6120E+02	-4.2551E-02	-3.3109E-01		
3	.1500	2.9785E-04	4.4418E-02	-6.8888E-03	-2.9147E-03	-4.2865E-03	1.2860E+00	-2.2920E-02	3.1590E+01
		7.2117E-01	-8.9186E-03	-1.6308E-01	1.5980E+02	-4.5378E-02	-3.3877E-01		
4	.2000	4.1508E-04	4.4558E-02	-3.7751E-03	-4.2830E-03	-9.7686E-04	1.9620E+00	3.9780E-01	3.1860E+01
		3.5552E-01	-2.3998E-01	6.5537E-02	1.5920E+02	-4.3563E-02	-3.4610E-01		
5	.2500	4.1508E-04	4.44[illegible]E-02	3.8362E-03	-2.9147E-03	-2.4155E-04	2.7760E+00	5.8470E-01	3.1470E+01
		2.0263E-01	-9.110[illegible]E-02	6.7028E-02	1.5860E+02	-4.0945E-02	-3.3807E-01		
6	.3000	5.1250E-04	4.4157E-02	1.0409E-02	-2.5726E-03	-1.7122E-03	2.6960E+00	7.4830E-01	3.1540E+01
		-1.2751E+00	1.6873E-01	-3.6303E-01	1.5830E+02	-3.8327E-02	-3.3353E-01		
7	.3500	9.9815E-04	4.4296E-02	1.4559E-02	-2.9147E-03	-6.0912E-04	2.2030E+00	4.9120E-01	3.2210E+01
		-1.2839E+00	-5.7805E-02	-6.6165E-01	1.5950E+02	-4.2970E-02	-3.4208E-01		
8	.4000	1.3472E-03	4.4418E-02	1.2484E-02	-2.5726E-03	-3.1835E-03	2.2720E+00	3.3930E-01	3.2410E+01
		-1.1069E+00	1.6397E-01	-1.6938E-01	1.5960E+02	-5.1225E-02	-3.2893E-01		
9	.4500	1.2308E-03	4.4698E-02	8.3322E-03	-3.2568E-03	-1.7122E-03	1.1830E+00	2.8070E-01	3.2690E+01
		-4.2167E-01	5.6985E-02	1.6598E-02	1.[illegible]230E+02	-4.7194E-02	-3.2812E-01		

Figure 4.5 INREAD Output

```
                                                                                                   *UERFIS PROGRAM*

----------------------------------------
PRIMARY AND SECONDARY STATES FILTER
----------------------------------------

XHAT (T+0)                        (T)
              1           2           3           4           5           6           7           8           9          10
    1    5.4000E-04  4.5500E-02  1.6250E+02 -8.1100E+00 -5.9000E+00 -1.4000E-02 -6.3000E-03  5.0000E-04  1.0690E+00 -3.2940E-01

             11          12          13          14
    1    1.2920E+00  3.0000E-01 -3.8000E-01  3.8000E-01

RHAT (T+0)                        (T)
              1           2           3           4           5           6           7           8           9
    1    0.          1.1000E-02  0.         -3.0000E-01  4.4000E-01  7.1000E-01  1.0000E+00  1.0000E+00 -1.0000E+00

IPOINT =     0
-------------

XHAT...ESTIMATES OF PRIMARY STATES   (T)
              1           2           3           4           5           6           7           8           9          10
    1    6.5655E-04  4.6045E-02  1.6225E+02 -7.7754E+00 -6.0392E+00 -9.9024E-03 -4.2727E-03 -1.3531E-03  3.2444E-01 -7.2655E-02

             11          12          13          14
    1    7.6279E-01 -1.2549E-01 -9.5390E-02  1.9580E-01

RHAT...ESTIMATES OF SECONDARY STATES (T)
              1           2           3           4           5           6           7           8           9
    1    1.2184E-04  1.1360E-02  9.4752E-05 -3.0000E-01  4.4000E-01  7.1017E-01  9.9917E-01  9.9993E-01 -1.0047E+00

PX...STATE COVARIANCE
              1           2           3           4           5           6           7           8           9          10
    1    2.0101E-06  3.5206E-14  6.5666E-09  1.8729E-06 -4.4848E-08  4.4728E-08  3.6087E-12  2.0650E-09  5.0815E-12 -4.4981E-09
    2    3.5206E-14  2.3040E-06 -1.8441E-06 -5.1076E-09  1.8272E-06 -1.3570E-12  4.4597E-08 -3.1809E-11  8.0311E-10 -1.6936E-11
    3    6.5666E-09 -1.8441E-06  1.1254E+00  4.3745E-03  3.0704E-03 -4.1897E-08 -2.2804E-06  1.8948E-06  5.5561E-04  1.3185E-05
    4    1.8729E-06 -5.1076E-09  4.3745E-03  1.2165E+00 -1.1475E-03  1.5776E-06  4.2697E-08 -1.8242E-05  2.1445E-06  3.6490E-03
    5   -4.4848E-08  1.8272E-06  3.0704E-03 -1.1475E-03  1.2162E+00 -2.1194E-06  1.8249E-05 -3.4284E-09  1.4312E-06 -2.1716E-06
    6    4.4728E-08 -1.3570E-12 -4.1897E-08  1.5776E-06 -2.1194E-06  9.9198E-07  2.6204E-11  2.0903E-11 -2.0572E-11  5.8968E-07
    7    3.6087E-12  4.4597E-08 -2.2804E-06  4.2697E-08  1.8249E-05  2.6204E-11  9.9187E-07  5.5118E-12 -1.3092E-09  1.4730E-10
    8    2.0650E-09 -3.1809E-11  1.8948E-06 -1.8242E-05 -3.4284E-09  2.0903E-11  5.5118E-12  9.9188E-07  1.1152E-09 -3.2973E-08
    9    5.0815E-12  8.0311E-10  5.5561E-04  2.1445E-06  1.4312E-06 -2.0572E-11 -1.3092E-09  1.1152E-09  9.3915E-03  6.4923E-09
   10   -4.4981E-09 -1.6936E-11  1.3185E-05  3.6490E-03 -2.1716E-06  5.8968E-09  1.4733E-10 -3.2973E-08  6.4923E-09  5.7083E-02
   11   -9.6060E-10 -2.2614E-09  2.0027E-05 -9.5129E-06  7.4083E-03 -1.5135E-08  6.6993E-08  1.9941E-11  9.1823E-09 -2.0995E-08
   12   -1.3109E-06  2.3150E-11 -1.0187E-07 -1.6600E-05 -6.9933E-06  5.3069E-07 -3.9767E-11  1.7165E-10 -4.9130E-11  4.7742E-08
   13   -4.1790E-11 -1.3067E-05  1.4605E-05  2.5769E-07 -1.1183E-04  2.6197E-10  5.2957E-07  3.4815E-11 -8.8234E-09  5.7409E-10
   14   -6.1640E-08  8.6957E-10  6.7392E-06  1.1787E-04 -4.5747E-08  2.1204E-10 -4.9356E-12  5.3371E-07 -1.8816E-09 -3.3087E-07
```

Figure 4.6 Intermediate Filter Printout

	11	12	13	14
1	-9.6060E-10	-1.3108E-05	-4.3390E-11	-6.1640E-08
2	-2.2614E-09	2.3150E-11	-1.1067E-06	8.4957E-10
3	2.0029E-05	-1.0189E-07	1.4605E-05	6.9392E-06
4	-9.5922E-06	-1.5600E-05	2.5768E-07	1.1287E-04
5	7.4083E-03	-6.9933E-06	-1.1183E-04	-4.5747E-08
6	-1.5135E-08	5.3069E-07	2.6197E-10	2.1204E-10
7	6.6798E-08	-3.3769E-11	5.2959E-07	-4.9354E-12
8	1.9941E-11	1.7165E-10	3.4815E-11	5.3171E-07
9	3.1971E-09	-4.9130E-11	-8.9234E-09	-1.9816E-09
10	-2.0915E-08	4.7742E-08	5.7409E-10	-3.3087E-07
11	1.4000E-01	2.2472E-08	6.6754E-07	-6.7924E-10
12	2.2472E-08	7.9635E-04	-3.9151E-10	1.6171E-09
13	6.6754E-07	-3.9151E-10	7.9634E-04	-4.4673E-11
14	-6.7924E-10	1.6171E-09	-4.4673E-11	8.0267E-04

PX...BIAS COVARIANCE

	1	2	3	4	5	6	7	8	9
1	5.0110E-05	0.	0.	0.	-3.2809E-07	0.	0.	0.	0.
2	0.	5.0804E-05	0.	0.	0.	-2.2655E-05	0.	0.	0.
3	0.	0.	9.9797E-06	0.	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.	0.	0.	0.	0.
5	-3.2809E-07	0.	0.	0.	1.0000E-03	0.	0.	0.	0.
6	0.	-2.2655E-05	0.	0.	0.	9.8957E-04	0.	0.	0.
7	0.	0.	0.	0.	0.	0.	9.9927E-04	0.	0.
8	0.	0.	0.	0.	0.	0.	0.	9.9998E-04	0.
9	0.	0.	0.	0.	0.	0.	0.	0.	2.0021E-04

IPOINT = 1

XHAT...ESTIMATES OF PRIMARY STATES (T)

	1	2	3	4	5	6	7	8	9	10
1	-5.4764E-05	4.6128E-02	1.6184E+02	-7.4793E+00	-5.7351E+00	-4.9728E-03	-4.7623E-03	-6.5307E-03	3.7766E-01	5.4339E-02

	11	12	13	14
1	4.9602E-01	6.5980E-02	-1.4375E-01	-8.8875E-02

RHAT...ESTIMATES OF SECONDARY STATES (T)

	1	2	3	4	5	6	7	8	9
1	3.5679E-04	1.1488E-02	2.6717E-03	-3.0000E-01	4.4000E-01	7.1023E-01	9.9445E-01	9.9998E-01	-1.0047E+00

PX...STATE COVARIANCE

	1	2	3	4	5	6	7	8	9	10
1	2.0111E-06	-1.2770E-14	9.6711E-09	3.5155E-06	-9.9235E-08	3.1478E-08	7.8643E-13	1.4376E-09	9.4512E-12	-1.0641E-08
2	-1.2770E-14	2.0033E-06	-3.4516E-06	-7.0152E-09	1.9126E-06	-6.3352E-13	3.1810E-08	-2.6846E-11	1.7836E-09	-5.3074E-11
3	9.6711E-09	-3.4516E-06	7.5039E-01	3.8895E-03	2.8399E-03	-2.7595E-08	-1.1652E-06	1.2340E-06	7.9164E-04	7.9420E-06
4	3.5155E-06	-7.0152E-09	3.8895E-03	8.3107E-01	-1.0707E-03	7.4923E-07	2.3733E-08	-1.2995E-05	1.0627E-06	5.0173E-03
5	-9.9235E-08	1.9126E-06	2.8399E-03	-1.0707E-03	8.3304E-01	-1.4463E-06	1.3003E-05	1.5413E-09	7.4025E-07	-7.1019E-07
6	3.1478E-08	-6.3352E-13	-2.7595E-08	7.4923E-07	-1.4463E-06	9.3997E-07	3.0297E-11	2.7137E-11	-2.7676E-11	8.2566E-09
7	7.8643E-13	3.1810E-08	-1.1652E-06	2.3730E-08	1.3003E-05	3.0297E-11	9.3982E-07	6.7349E-12	-1.0661E-09	2.3270E-10
8	1.4376E-09	-2.6846E-11	1.2340E-06	-1.2995E-05	1.5413E-09	2.7137E-11	6.7349E-12	9.3971E-07	1.4421E-09	-4.6191E-08
9	9.4512E-12	1.7836E-09	7.9164E-04	1.0627E-06	7.4025E-07	-2.7676E-11	-1.0661E-09	1.4421E-09	9.4177E-03	1.3770E-08
10	-1.0641E-08	-5.3074E-11	7.9420E-06	5.0173E-03	-7.1019E-07	8.2566E-09	2.3270E-10	-4.6191E-08	1.3770E-08	5.9151E-02
11	-1.2641E-09	-4.1612E-09	1.4492E-05	-1.1860E-05	1.1064E-02	-2.1111E-09	1.0271E-07	5.0886E-12	1.6456E-09	-3.8677E-08
12	-7.9961E-07	-5.5013E-11	-3.4476E-08	-1.0251E-05	-2.4574E-06	5.1841E-06	[illegible].1767E-10	2.4880E-10	-2.5304E-10	8.4637E-08
13	2.7317E-11	-7.3525E-07	1.3515E-06	1.6110E-07	-4.7032E-05	3.9477E-10	5.3422E-06	6.8335E-11	-1.9202E-09	2.2013E-07
14	-1.9955E-08	4.8756E-10	[illegible]	4.7760E-05	2.0625E-09	1.5669E-10	5.1897E-11	5.3994E-06	1.0591E-04	-5.9610E-07

Figure 4.6 (Concluded)

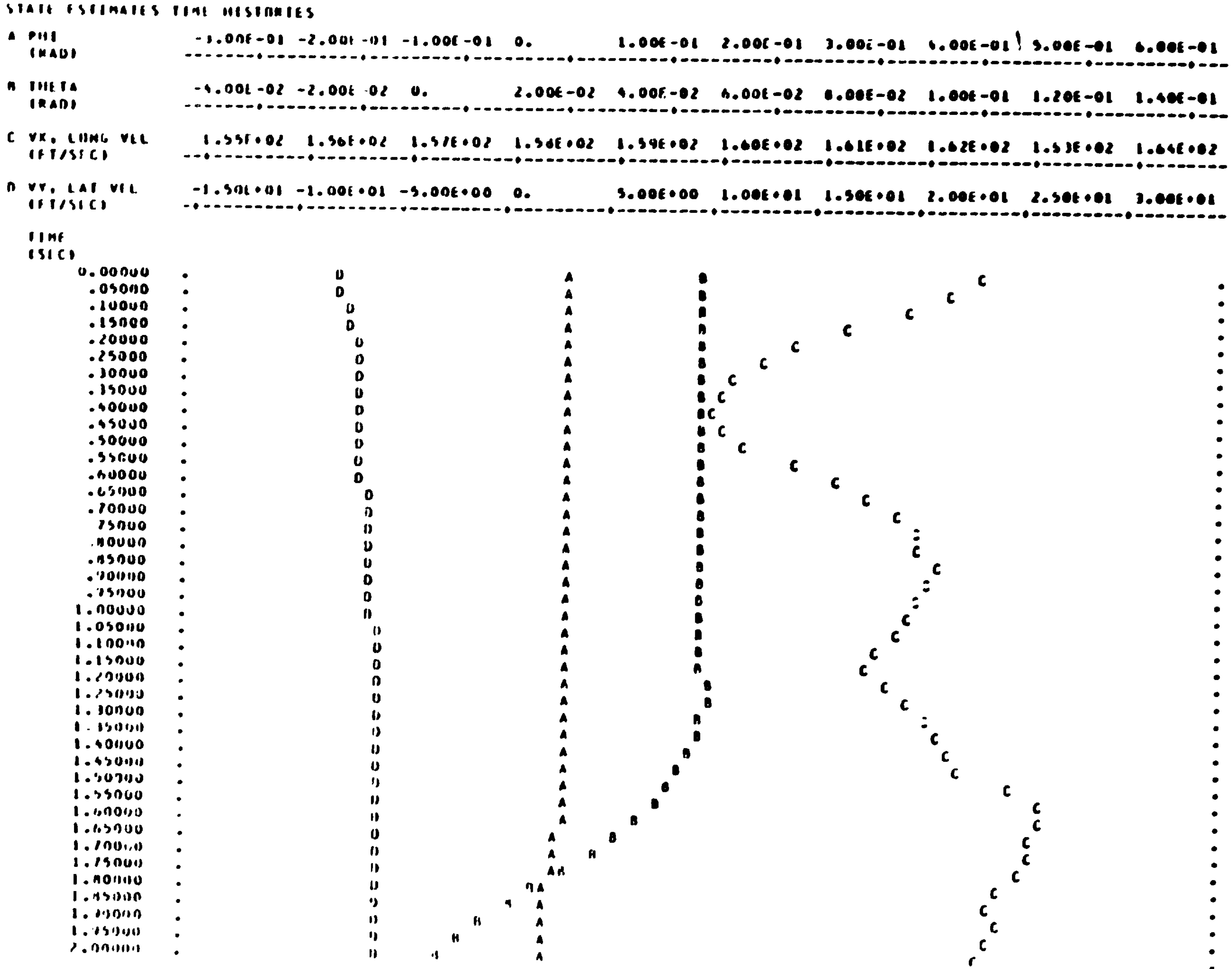


Figure 4.7 Printer Plots

where, TITLE – is the title information

VECTOR – is a vector of either $\hat{\underline{x}}_1$, $\dot{\hat{\underline{x}}}_1$, $\hat{\underline{x}}_2$ or $\hat{\underline{w}}$

M – is the length of the vector

NDPF – is the number of data points on each file section

K – is a dummy integer variable

V. EFFECTIVE USE OF THE DEKFIS PROGRAM

The performance of the DEKFIS program is dependent on the proper choice of the initial conditions for the primary and secondary states ($\underline{x}_o$ and $\underline{b}_o$), the initial covariances associated with the primary and secondary state initial condition estimates ($\underline{P}_x$ and $\underline{P}_{b_o}$), and the process and measurement noise covariance matrixes ($\underline{Q}$ and $\underline{R}$). The following systematic procedure has been developed for determining these program inputs:

1. Compute measurement noise covariance matrix-$\underline{R}$. Assume $\underline{R}$ is diagonal and calculate for each measurement. This can be done approximately by inspecting the measurement time history and picking out the peak-to-peak value of the zero mean gaussian measurement noise on that channel. Assume this peak-to-peak value $\cong 6\sigma$

2. Estimate values of biases and scale factors for each of the measurements. Sometimes these are readily apparent. If they are not known, set the biases equal to 0.0 and the scale factors equal to 1.0.

3. Estimate the initial covariance (P_{b_o}) for each of the biases and scale factors to be identified (i.e., treated as secondary states). Assume P_{b_o} is diagonal and calculate each variance term (σ^2), keeping in mind that 6 is the total peak-to-peak excursion allowable.

4. Estimate the initial conditions on the states. Using a best estimate for the measurement at t=0 (usually the initial data point is as good as any) and the bias and scale factor estimates determined in step 2, back calculate through the measurement equations to estimate the states at t=0.

5. Estimate the initial covariance (P_{x_o}) for the initial conditions on the states. This is possible by taking the measurement noise covariance from step 1 and covariances for the biases and scale factors from step 3 and using the measurement equations for back calculation.

6. Estimate the process noise covariance matrix - $\underline{Q}$. Assume $\underline{Q}$ is diagonal. The estimate for $\underline{Q}$ is the most difficult to determine, and several runs may be required to give

> **satisfactory results. A low value for a given diagonal element of $\underline{Q}$ will result in a false convergence of the states, and the measurement matches will show discrepancies. A large value for a given element of $\underline{Q}$ will lead to unconverged states, and the update correction in the predict/update cycle of the Kalman filter will be large. In other words, the measurements will drive the filter leading to deceivingly good measurement matches. It is necessary to find a value for $\underline{Q}$ which will give good measurement matches, but also will be converged.**

This procedure was used in the validation of the rotor state estimator. One example case was run assuming only blade flapping degrees of freedom. Lag degrees of freedom were omitted by simply setting input flags. Thus, the state variables were:

$$\underline{x}^T = [\beta_o, \beta_{1c}, \beta_{1s}, \psi_r, \dot{\beta}_o, \dot{\beta}_{1c}, \dot{\beta}_{1s}, \dot{\psi}_r]$$

The example was for a three-blade rotor, and the measurement variables consisted of the flap angle of each blade and the cosine of the rotor azimuth angle; i.e.,

$$y_i = \beta_i = \beta_o - \beta_{1c}\cos(\psi_r + \phi_i) - \beta_{1s}\sin(\psi_r + \phi_i) \ , \qquad i = 1,2,3$$

$$y_4 = \cos\ (\psi_r)$$

where the blade spacing angles, ϕ_i, are known. The measurements were corrupted by relatively small amounts of white noise.

The inputs to the filter were the state initial conditions, x_o, the initial state covariance matrix, P_o, the measurement noise covariance, R, and the process noise covariance, Q. When processing actual flight data, x_o, P_o and R are reasonably well known. Because the performance of the filter largely depends on the ratio of the norms of P_o, R, and Q, it is the

choice of the less-well-known Q which determines the performance. The rotor estimator is simply:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} W$$

where

$$E\,[w(t_i)] = 0, \quad E[w(t_i)\,w(t_j)^T] = Q\delta_{ij}$$

and (in this case)

$$\underline{x}_1^T = [\beta_o, \beta_{1c}, \beta_{1s}, \psi_r] \quad .$$

It can be seen that setting Q to zero tells the filter that the states are constant and that letting Q approach infinity (relative to R) tells the filter to weight the measurements very heavily. When estimating trim tip-path-plane angles and instrumentation biases, Q may be set to zero. When the tip-path plane is not constant due to a control input, an intermediate value of Q is desirable. Too large a value for Q results in the state estimates being heavily driven by the measurements.

The effect of Q on the rotor state estimator was investigated for this example problem. Figure 5.1 shows a portion of the measured and estimated time histories of the flapping angle of the first blade from a run for which Q was set to zero. A control input beginning at $t = 2.5$ seconds causes a large change in the blade's flapping response, but this change is not tracked by the filter. Figure 5.2 is the same plot from a run for which Q was set to 10^8. Here, as expected, the measurement was tracked exactly (the symbol "B" overwrote the symbol "A" everywhere).

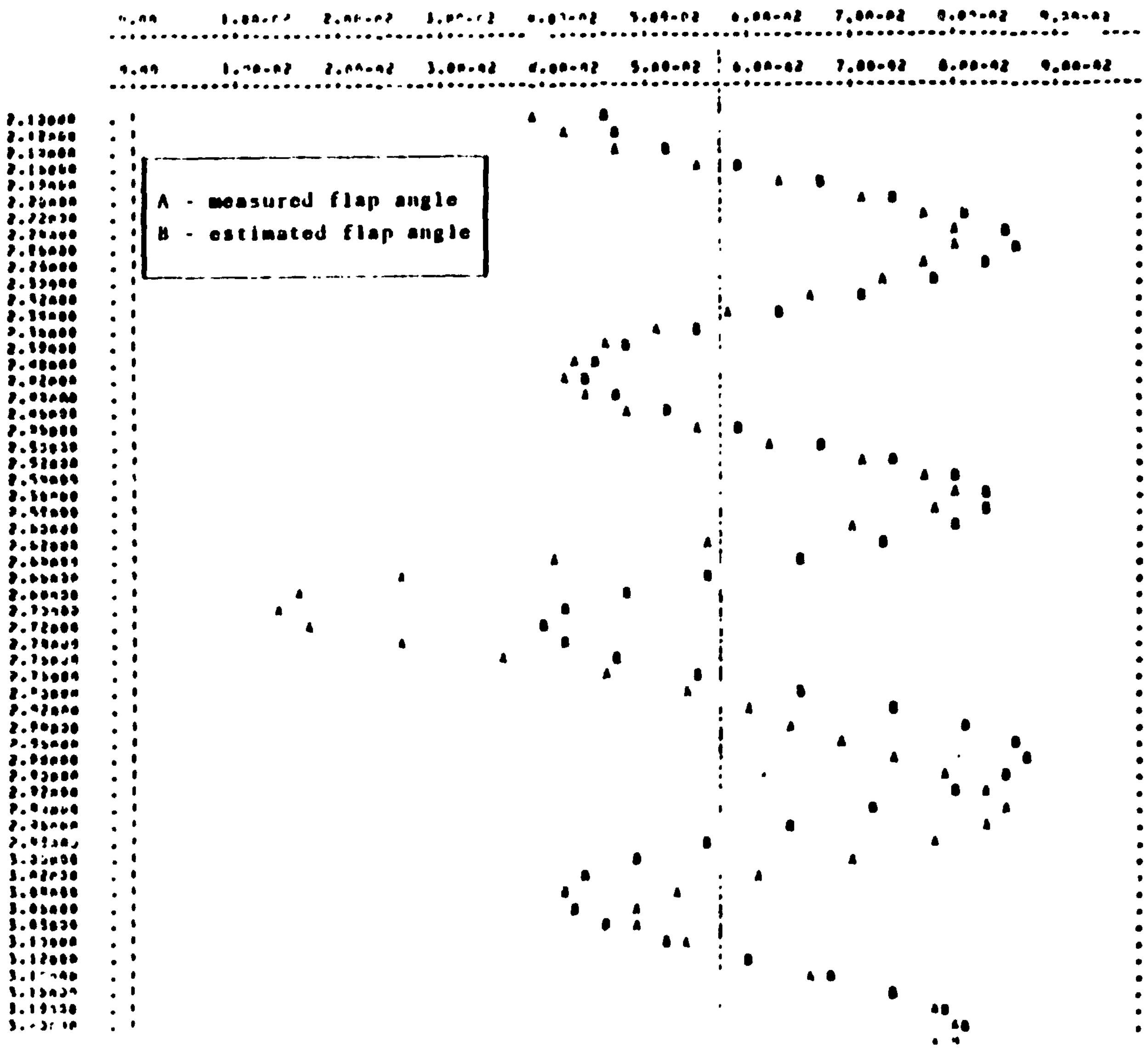


Figure 5.1 Rotor State Estimator Example with Q = 0

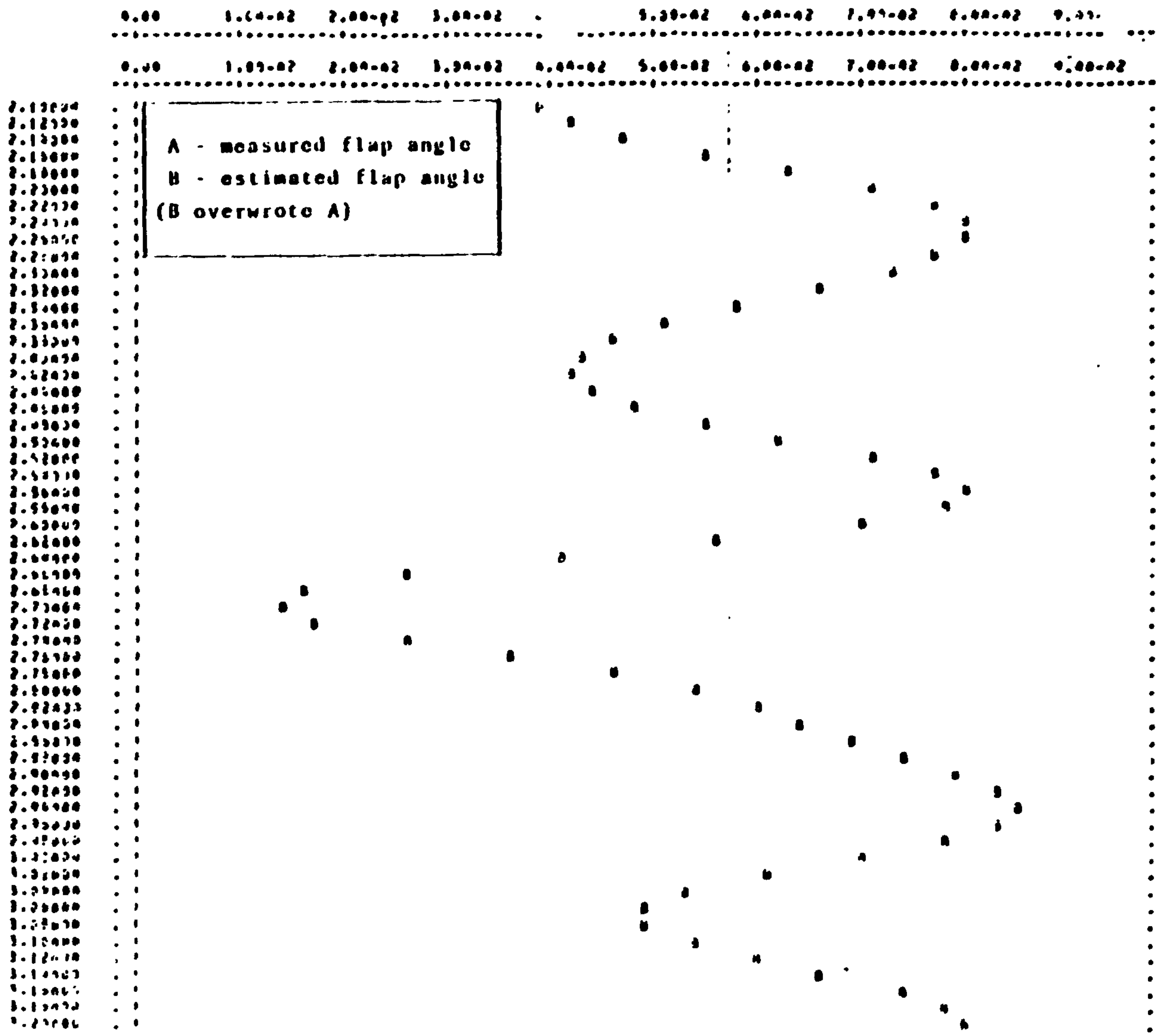


Figure 5.2 Rotor State Estimator Example with $Q = 10^8$

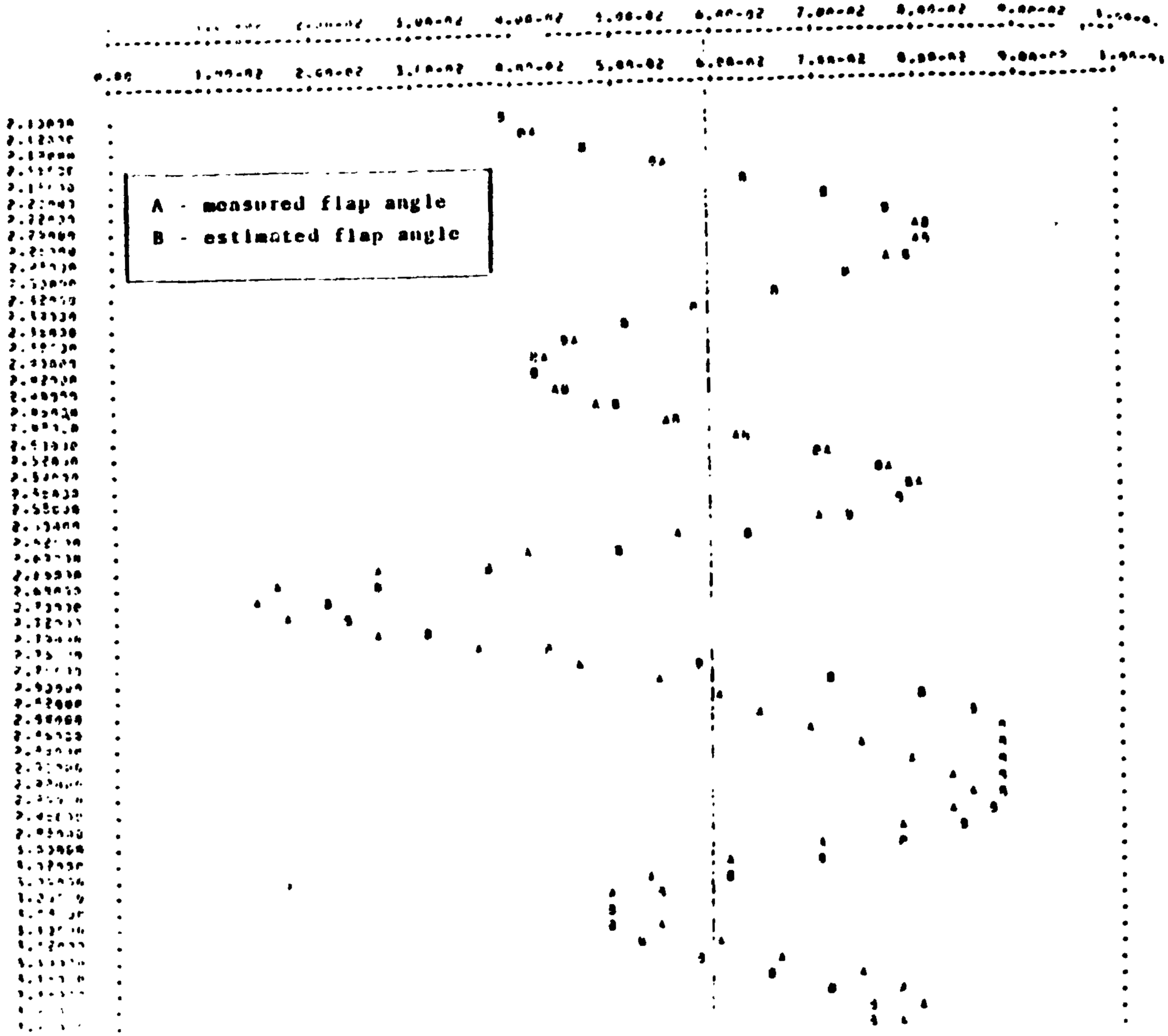


Figure 5.3 Rotor State Estimator Example with Q = 1

Figure 5.3 is the same plot from a run for which Q was set to 1.0, and it can be seen that the measurement is partially tracked. A somewhat higher value of Q would be required for satisfactory results.